Low Velocity Impact of a Deformable Multi-Body System

Jose M. Goicolea and Francisco J. Martínez

E.T.S. Ingenieros de Caminos, Universidad Politécnica de Madrid, Ciudad Universitaria, 28040 Madrid, Spain.

ABSTRACT

An analysis methodology for impact of multi-body systems comprising deformable and rigid bodies is described, and applied to the analysis of a portable radar system which undergoes elastic and plastic deformation as well as significant rigid body motion during impact. A detailed local finite element analysis is first performed for developing simplified non-linear structural properties, followed by a global finite element analysis which includes treatment of rigid bodies through the discrete equations of motion. Certain parameters, such as the flexibility of structural members in the load path, and the characteristics of the impact pad, were found to be essential for a correct simulation.

INTRODUCTION

Low velocity (e.g. in the order of 10 m/s) impact of mechanical equipment may involve both elastic-plastic deformation of some parts, as well as significant rigid body motion. Some of the parts in the load path near the contact zone may develop significant elastic and plastic deformations, while others will undergo essentially a rigid body motion. In this latter case, a detailed finite element model would result in excessive costs from several points of view: the preparation and checking of the model, the analysis on the computer, etc.

For an appropriate representation of the impact dynamics and energy dissipation characteristics, it is important to model correctly the elastic-plastic flow for the parts undergoing significant deformation. As regards the rigid parts, it is necessary to represent adequately the rigid body dynamics, as a substantial interchange may take place between "translational" and "rotational" kinetic energy terms.

Traditionally, the fields of finite elements and rigid body dynamics have remained separate and unconnected. The finite element techniques evolved from the early works of Clough [4] for linear structural applications, to complex nonlinear codes such as DYNA3D [13] or ABAQUS [10]. In the multibody kinematics and dynamics field, the evolution has been favoured by the automotive, robotics and space applications, producing codes such as ADAMS [9], DADS [7], or COMPAMM [5]. Recently, some authors have developed methods which combine both disciplines [12, 1, 3].

In this paper, the impact analysis of a portable radar equipment is performed employing a methodology based on detailed local elastic-plastic analysis, followed by a global analysis coupling nonlinear structural members and rigid bodies. The first analysis is achieved with the program ABAQUS [10], and the second with DYNA3D [13]. First the methodology and formulation for rigid body analysis is summarised briefly. Following, the results for both analyses are presented and discussed.

METHODOLOGY

The problem under consideration was to explain the impact behaviour of a portable radar equipment. It had been observed that, subjected to an impact from a free fall in the field, the axle governing the elevation of the antenna was damaged with severe plastic deformation, inhibiting further functionality. The rest of the equipment, including the antenna, withstood the impact without severe damage.

The methodology followed comprised two stages. In the first stage, a detailed elastic-plastic local analysis of the elevation axle was performed. This allowed the determination of a nonlinear resultant law for moment/rotation (M/θ), to be used in a structural representation of the axle within a global model.

The global analysis was performed with the explicit dynamics code DYNA3D [13]. In this analysis, only the elements in the load path between the contact zone at the tip of the antenna and the elevation axle were modelled as deformable, using structural elements (beams). The antenna itself was modelled as elastic, while the portion of the elevation axle wich failed was modelled as elastic-plastic following the resultant M/θ law derived from the local analysis. Impact was modelled against a contact pad, as a rigid stonewall was considered unrealistic.

The rest of the equipment was modelled as a single rigid body, linked rigidly to the axle of the antenna. This rigid zone held 95% of the mass of the equipment, and thus contained all the significant kinetic energy.

During impact, the initial kinetic energy will be transformed or dissipated into elastic (stored) deformation energy (W^{el}), plastic (dissipated) deformation energy (W^{pl}), kinetic energy of translation ($\frac{1}{2}MV_G^2$), where M is the mass of the body and V_G the velocity of its center of mass G, and kinetic energy of rotation ($\frac{1}{2}\boldsymbol{\Omega} \cdot \boldsymbol{J}_G \cdot \boldsymbol{\Omega}$), where $\boldsymbol{\Omega}$ is the angular velocity and $\boldsymbol{J}_G$ the inertia tensor at G.

FORMULATION FOR FINITE ELEMENT MULTI-BODY ANALYSIS

The finite element formulation employed is well known [6, 10] and need not be repeated here. However, the treatment of rigid bodies within a finite element context is not so widespread. For the sake of completeness, a brief summary of this formulation will be given here. Additional details may be found in the paper by Benson and Hallquist [2].

The inertial properties of a rigid body $\mathcal{B}$ are described by its mass and central inertia tensor,

$$\begin{aligned} M &= \int_{\mathcal{B}_0} \rho_0 \, dV \\ \boldsymbol{J}_G &= \int_{\mathcal{B}_0} (X^2\mathbf{1} - \boldsymbol{X} \otimes \boldsymbol{X}) \rho_0 \, dV \end{aligned}$$

where ρ_0 is the mass density in the reference configuration $\mathcal{B}_0$, $\boldsymbol{X}$ the vector for a generic particle, $\mathbf{1}$ the unit tensor, and $\otimes$ denotes diadic (tensor) product.

It is convenient to describe the motion of $\mathcal{B}$ using "quasi-velocities" Q_i [8], whose relation with the generalised coordinates q_i is

$$Q_i = \psi_{ij}(\boldsymbol{q})\dot{q}_j.$$

The following choice of "quasi-velocities" is recommended:

- V_i, components of $\boldsymbol{V}_G$ in global (inertial) axes,
- Ω_i, components of $\boldsymbol{\Omega}$ in global axes,
- Additional modes η_i may be included to represent deformation of the body, which could be nodal velocities or rates of (modal) normal coordinates.

The generalised forces for rigid bodies corresponding to this choice of coordinates are

$$F_i^X = \int_{\mathcal{B}_0} \rho_0 f_i \, dV + \int_{\partial\mathcal{B}_0} t_i \, dS + \sum_k f_{ki}^* + \left\langle \lambda_k, \frac{\partial \phi_k}{\partial V_i} \right\rangle; \qquad (1)$$

$$F_i^{\Omega} = \int_{B_0} \rho_0 f_j R_{jn} e_{nim} X_m \, dV + \int_{\partial B_0} t_j R_{jn} e_{nim} X_m \, dS + \sum_k f_{kj}^* R_{jn} e_{nim} X_m^* + \left\langle \lambda_k, \frac{\partial \phi_k}{\partial \Omega_i} \right\rangle; \tag{2}$$

where f_i are the body forces; t_i the tractions at the boundary; f_{ki}^* the concentrated forces; λ_k the Lagrange multiplier for constraint ϕ_k, $\langle \cdot, \cdot \rangle$ being the inner product for each constraint; R_{jn} is the rotation matrix, and e_{nim} the permutation operator. The resulting Lagrange equations of motion, in vector form, are

$$M \dot{\boldsymbol{V}}_G = \boldsymbol{F}^X; \tag{3}$$

$$\boldsymbol{J}_G \cdot \dot{\boldsymbol{\Omega}} + \boldsymbol{\Omega} \times (\boldsymbol{J}_G \cdot \boldsymbol{\Omega}) = \boldsymbol{F}^{\Omega} \tag{4}$$

The expression for the inertia tensor in a global fixed frame must take into account the rotation $\boldsymbol{R}$. This is achieved incrementally with the algorithm of Hughes-Winget [11]:

$$\boldsymbol{R}^{n+1} = \Delta \boldsymbol{R}^{(1)} \cdot \Delta \boldsymbol{R}^{(2)} \ldots \cdot \Delta \boldsymbol{R}^{(n+1)}, \tag{5}$$

with

$$\Delta \boldsymbol{R}^{(n+1)}(\Delta \boldsymbol{\theta}^{(n+1)}) = \boldsymbol{1} + (\boldsymbol{1} - \frac{1}{2} \Delta \hat{\boldsymbol{\theta}}^{(n+1)})^{-1} \cdot \Delta \hat{\boldsymbol{\theta}}^{(n+1)} \tag{6}$$

where $\Delta \boldsymbol{\theta}^{(n+1)} = \boldsymbol{\Omega}^{n+1/2} \Delta t$, and $\Delta \hat{\boldsymbol{\theta}}$ is the skew-symmetric matrix for $\Delta \boldsymbol{\theta}$ ($\Delta \hat{\theta}_{ij} = e_{ikj} \Delta \theta_k$). The inertia tensor is updated incrementally via

$$\boldsymbol{J}_G^{n+1} = \Delta \boldsymbol{R}^{(n+1)} \cdot \boldsymbol{J}_G^n \cdot \Delta \boldsymbol{R}^{T(n+1)}. \tag{7}$$

Finally, equations (3, 4) are integrated in time with an explicit central difference scheme, to update velocities and coordinates.

NUMERICAL RESULTS

Local Elastic-Plastic Analysis

In order to evaluate the failure mechanism and strength of the axle, a detailed elastic-plastic analysis was conducted. The model consisted of the portion of the axle in contact with a steel block, which was inserted into it and forced it into torsion. The backplane of the block was given an imposed finite rotation, which was transmitted to the axle by a contact "slidesurface" (figure 1).

The properties of the material were those of AISI-304A steel, $E = 200\,\text{GPa}$, $\nu = 0.33$, $\sigma_y = 290\,\text{MPa}$. The model was achieved with 2946 elements of type C3D8 using program ABAQUS [10].

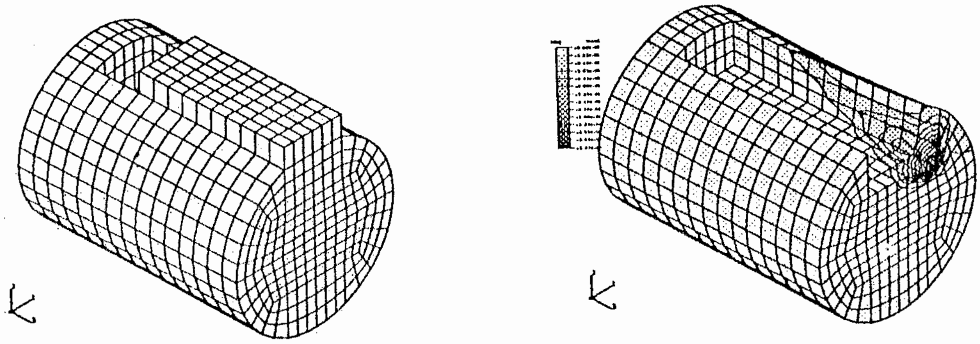

Figure 1: *Local model for failure of elevation axle*

For an imposed rotation of $\theta = 0.4\,\text{rad}$, the analysis was carried out using 17 load increments. Failure of the axle was observed with substantial plastic deformations of a wedge-shape portion (max $\bar{\varepsilon}_p = 1.57$) (figure 1). This predicted failure mode coincided very closely with the observed one. The resultant moment–rotation law is depicted in figure 2.

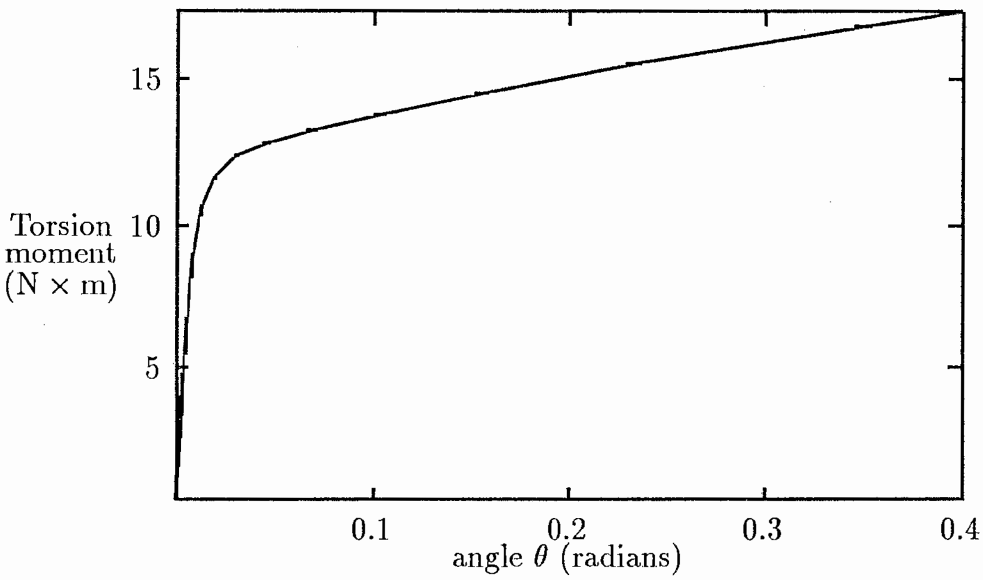

Figure 2: *Resultant M/θ law for axle failure*

Global Impact Analysis

A global impact analysis of the equipment was performed, representing a fall with 2 m/s vertical velocity as well as rotation, with the antenna end hitting the ground (figure 3). The antenna and the elevation axle connecting it to

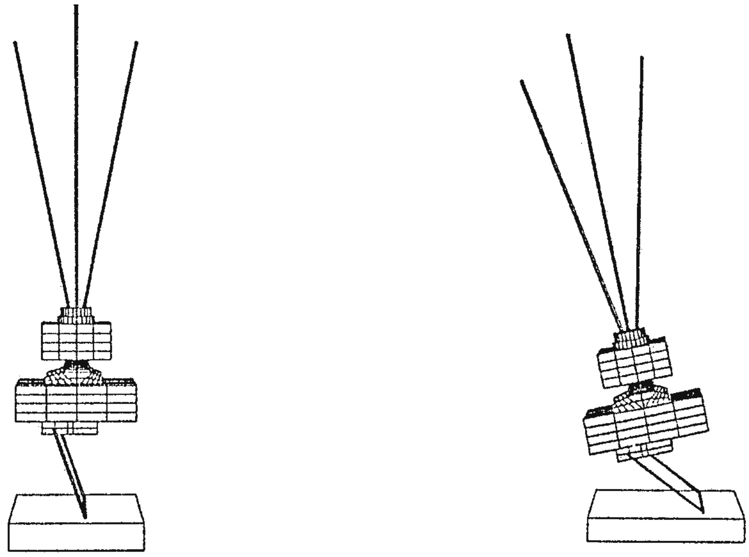

Figure 3: *Configuration of model before and after impact*

the rest of the equipment were represented as deformable structures with beam elements, assigning realistic values to their flexibility. The portion of the axle susceptible of failure was assigned elastic-plastic properties for the resultant M/θ relation obtained in the local analysis. The rest of the equipment was represented as a single rigid body, connected to the elevation axle. The mass of the equipment was 28 kg.

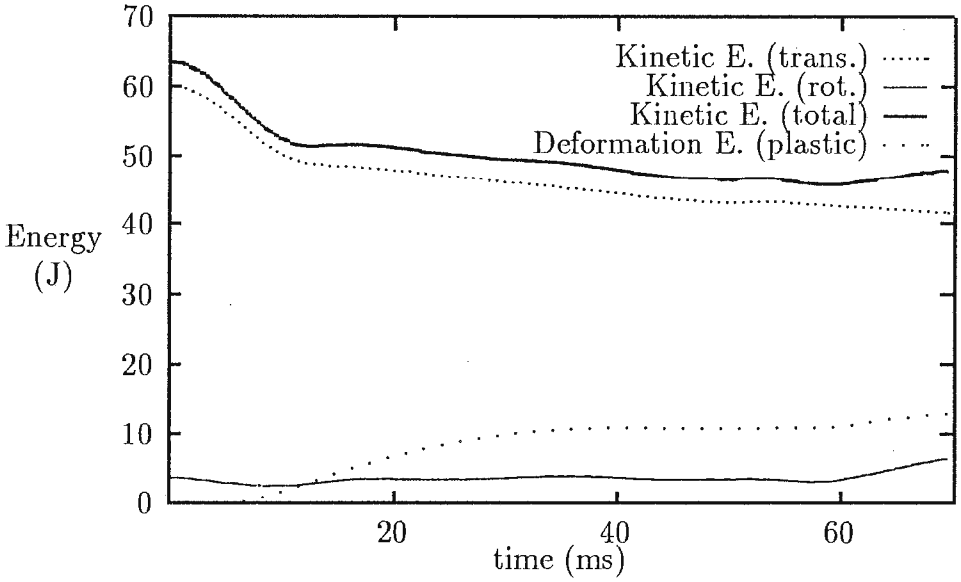

Figure 4: *Time histories of energy terms*

A ground pad was used in order to provide a realistic representation of the contact with the ground. This pad allowed for an initial penetration of

10 mm with a very low resistance; subsequently it behaved as elastic up to a strength of 1.2 kN, representative of the failure of a 13.5 mm wide strip of hard soil.

The model was performed using DYNA3D, employing 7643 explicit time-steps for a total simulation time of 70 ms. An overall view of the model before and after impact is shown in figure 3. After impact the model rebounds, still holding a substantial kinetic energy.

The histories of energy components are shown in figure 4. One can notice the transformation between translation kinetic energy and rotation kinetic energy, as well as plastic deformation energy at the axle. An inspection of the plot of torsion moment vs. time in the axle (figure 5) shows it achieved the elastic limit and subsequently deformed plastically until failure.

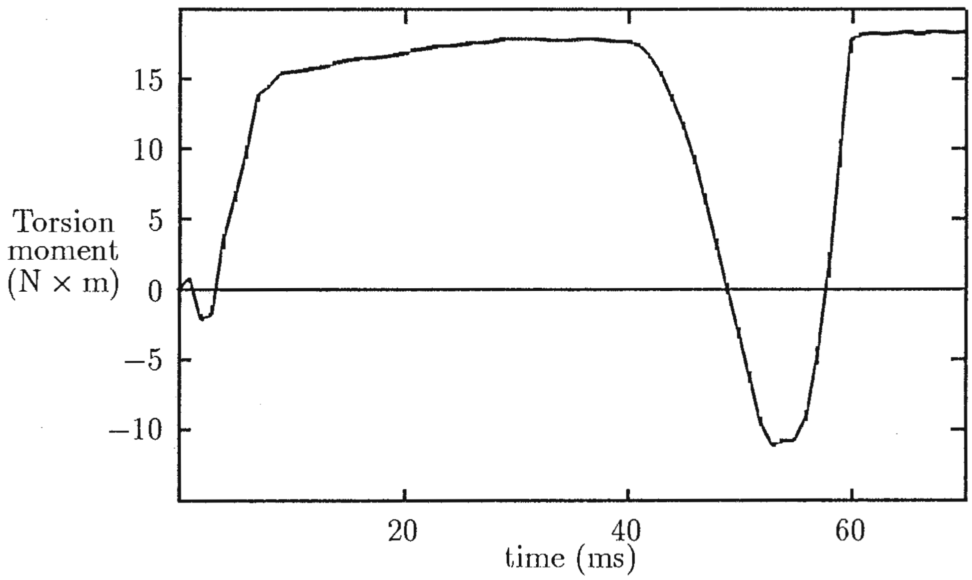

Figure 5: *Time history of torsion moment in axle during impact*

CONCLUDING REMARKS

With the methodology proposed in this work it is possible to perform realistic simulations with a low computer cost for the low-velocity impact of complex mechanical equipment, using a combination of rigid body dynamics, flexible bodies, and elastic-plastic bodies.

REFERENCES

1. Avello, A. "Dynamics of flexible mechanisms with cartesian coordinates and large deformation theory." *Diss.*, Univ. de Navarra (Spain) (1990).

2. Benson, D.J. and Hallquist, J.O. "A simple rigid body algorithm for structural dynamics programs." *Int. J. Num. Meth. Eng.* **22**, 723–749 (1986).
3. Cardona, A. and Géradin, M. "Time integration of the equations of motion in mechanism analysis." *Comp. & Struct.* **33**, 801-820 (1989).
4. Clough, R.W. "The finite element in plane stress analysis." *Proc. 2nd ASCE conf. electronic computation,* Pittsburg, Pa. (1960)
5. García de Jalón, J. et al. *COMPAMM: Computer analysis of machines and mechanisms,* program documentation, CEIT, San Sebastian, Spain (1989).
6. Hallquist, J.O. "Theoretical manual for DYNA3D." *Rept. UCID-19401,* Univ. of California, Lawrence Livermore National Laboratory (1982).
7. Haug, E. *DADS program documentation,* Center for computer aided design, U. of Iowa (1989)
8. Meirovitch, L. *Methods of Analytical Dynamics,* McGraw-Hill, New York (1970).
9. Orlandea, N. "Node-analogous sparsity-oriented methods for the simulation of mechanical systems." *Diss.*, Univ. of Michigan (1973).
10. Hibbit, Karlsson & Sorensen. *ABAQUS User's manual.* HKS, Inc. (1989).
11. Hughes, T.J.R. and Winget, J. "Finite rotation effects in numerical integration of rate constitutive equations arising in large-deformation analysis." *Int. J. Num. Meth. Eng.* **15**, 723–749 (1980).
12. Simo, J.C. and Wong, K.K. "Unconditionally stable algorithms for rigid body dynamics that exactly preserve energy and momentum." *Int. J. Num. Meth. Eng.* **31**, 19–52 (1991).
13. Whirley, R.G. and Engelmann, B.E. "DYNA3D, a nonlinear, explicit three-dimensional code for solid and structural mechanics. User's manual." *Rept. UCRL-MA-107254,* Univ. of California, Lawrence Livermore National Laboratory (1993).