NUMERICAL SIMULATION OF IMPACT PROBLEMS

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1. INTRODUCTION

Impact problems are among the more demanding ones in terms of computer time. In comparison with other mechanical or structural problems, the high demands in computer time are due to a number of characteristics which generally accompany impact problems, namely: they are dynamic, non-linear and, very often, require a full three-dimensional treatment.

Their dynamic character implies that the inertia terms of the equations of motion cannot be neglected in the analysis; furthermore, not only rigid body inertias (which can be represented as body forces), but also local inertias (as activated in wave propagation phenomena) must be duly accounted for.

Impact problems are non-linear because of a variety of reasons. First of all, the contact conditions between the impacting bodies are inherently non-linear. Furthermore, the levels of deformations induced in one or both of the bodies are usually sufficient to develop non-linear mechanical behaviour in the materials (constitutive non-linearity) and even to require the use of non-linear measures for the deformation process (geometric non-linearity).

Occasionally, the symmetries of the problem allow the use of idealizations with less than three space dimensions, chiefly two-dimensional axisymmetric models. However, this is applicable only in normal impacts between aligned bodies with axially symmetric geometries and in the absence of transverse forces. Although one of the three cases presented here falls in this category, the majority of cases require a full three-dimensional analysis in order to be reasonably representative of the real problem under consideration.

2. BASIC FORMULATION

Two basic sets of equations govern the mechanical behaviour of deforming bodies: the equations of motion and the constitutive laws.

The local formulation of the equations of motion is as follows:

$$\sigma_{ij,j} + \rho (f_j - \ddot{u}_j) = 0$$  \hspace{1cm} (1)

where \(\sigma_{ij}\) is the Cauchy stress tensor
\(\rho\) is the density
\(f_j\) is the body force
\(\ddot{u}_j\) is the local acceleration

In their numerical implementation, these equations are referred to a grid of small elements spanning the bodies under consideration. In a Lagrangian
grid, which deforms with the body, the equations of motion can be expressed for each nodal point in the grid as:

\[ \ddot{u}_j = \left( \int \sigma_{ij} n_i \, ds + f_j \right) / m \]  \hspace{1cm} (2)

where the integral extends to the surface surrounding the volume of material whose mass \( M \) is considered to be concentrated at the node. Since grid moves with the material, the mass associated to each node does not change with time.

The constitutive laws express the relationship between the states of stress and strain at each point in the body. Due to the lack of objectivity of the traditional time derivatives of Eulerian tensors, the formulation of the constitutive laws is not trivial, even for simple material behaviours, whenever large strains and rotations may be involved. The more common procedure is based on rate-type formulations:

\[ \dot{\sigma}_{ij} = F(D_{kl},...) \]  \hspace{1cm} (3)

where \( \dot{\sigma}_{ij} \) is an objective rate (Jaumann, Truesdell, Lie, etc.) of the Cauchy stress tensor
\[ D_{kl} = (\ddot{u}_{k,l} + \ddot{u}_{l,k})/2 \] is the rate of deformation tensor
\( F \) is a scalar or tensorial function describing the relationship between the stress rate and the deformation rate and possibly other variables such as the temperature, accumulated plastic strain, etc.

In a similar role to the constitutive laws, impact problems generally require the use of contact laws. Just like the constitutive laws relate relative motions inside a body to its internal state of stress, contact laws relate relative motions across a contact interface to the forces of interaction developed at that interface.

For impact problems in which relative particle velocities are well below the speed of sound of the material, the most convenient formulation is afforded by explicit Lagrangian procedures. The scheme of the calculations is depicted in Fig. 1.

![Fig. 1 Computational cycle](image-url)
The integration cycle shown in the figure must be repeated as many times as necessary to span the time interval of interest. The basic limitation imposed on the size of the time step for integration is given by the Courant condition; this condition essentially requires that the time step be so short that no two nodes have time to communicate during one time step, which effectively uncouples their equations of motion. In most cases, this limitation is also sufficient to ensure adequate accuracy of the integration process.

3. COMPUTATIONAL CONSEQUENCES

The previous scheme presents great advantages for its numerical implementation. Among them:

- all equations of motion are uncoupled
- no matrices need to be inverted; no systems of equations have to be simultaneously solved
- the constitutive laws can be used directly in an explicit form; no special treatment is required for material softening and similar behaviours, which inevitably lead to ill-conditioning of matrices in other formulations
- storage and computer time requirements increase only linearly with the size of the problem (number of elements in the grid and number of time steps in the time interval of interest)
- most of the operations are easily amenable to natural implementations in vector and parallel processors; the main exception relates to the contact algorithms, which by necessity involve searches to find out which element is contacting which at each time.

There are certain limitations, however, in this procedure. The main ones are listed below:

- the conditional stability of the explicit integration process may require large numbers of time steps for solving certain problems, but this is seldom of great concern in impact cases, which are typically short-lived events
- contact searches are often time consuming and, as mentioned earlier, awkward to vectorize or parallelize
- special measures are required when relative particle velocities approach the velocity of sound and, generally, for the control of shock waves in the material

Overall, it can be concluded that explicit Lagrangian procedures are very well suited to deal with impact problems within the conditions mentioned before. This is corroborated by the examples discussed in the following sections.

4. LOW-LEVEL WASTE CONTAINER

The example presented here has been taken from a series of analyses performed in order to assess the feasibility of achieving a safe and economically attractive transport for a given waste package. The motivation for the analyses was that certain low-level nuclear waste contained in glass bottles had to be transported. An attractive alternative was to surround the glass bottle with a shock absorbing material and to enclose
this assembly within a steel drum.

The investigations conducted were designed to assess the feasibility of the proposed concept and to optimize the properties of the shock-absorbing material. The design impacts for the assembly were the 9m drops onto an unyielding target originally proposed in 1973 by the International Atomic Energy Agency [3].

The program selected for the calculations was DYNA3D [1]. Shell elements were used to model the steel drum and the glass bottle. Solid elements were employed for the shock absorbing material as well as for the waste inside the bottle. The target was represented with a rigid stonewall.

Although different models were created for studying impacts with different symmetries, the model shown in Fig. 2, composed 576 shell elements and 1350 hexahedral solid elements, makes use of only one plane of symmetry. For scale, the height and diameter of the steel drum are 0.84m and 0.57m, respectively.

![Fig. 2 Mesh used to model the skel drum, shock-absorbing material, glass bottled and radioactive contents](image)

As for the behaviour of the various materials, the contents of the bottle were assigned water properties. Glass was assumed to behave as an elastic-brittle material. The steel drum was represented with an elastic ideally plastic material with a von Mises yield surface. A similar type of constitutive behaviour, albeit with different properties, was assigned to the shock-absorbing material. In this latter case, preliminary hand calculations were used to provide initial estimates for the desirable properties, later followed by more detailed numerical analyses.

Figures 3 and 4 show some results corresponding to one of the more
demanding cases: a corner impact following a free drop from 9m, in pure
translation and with the centre of gravity directly above the impact point.
The problem is made harder, even in physical terms, because of the very
large deformations which necessarily ensue an energetic corner impact.
Numerically, additional demands arise from the fairly large duration of the
problem (much longer than a base or side impact); the long duration is
associated to the relative softness of an impact in which the contact area
is growing slowly from an initial zero value.

Fig. 3 Contours of equal plastic strain (A=0.1, B=0.2, C=0.3, D=0.4, E=0.5)

Fig. 4 Contours of equal effective stress in the glass bottle at peak
values (A=10MPa, B=20MPa, C=30MPa, D=40MPa, E=50MPa)

Fig. 3 shows values of plastic strains occurring in the shock absorbing
material (the steel drum, the glass bottle and its contents have been
removed in this plot) at the end of the impact event. Fig. 4 presents
contours of effective stresses in the glass bottle at the time when maximum
values are reached.

The number of time steps used for integration in this analysis was on the
order of 5,000. The Convex C-220 utilized under 15 CPU minutes in
completing the calculations.

The analyses conducted finally allowed demonstrating the feasibility of the proposed concept and determining the properties of a material which would ensure physical separation between bottle and drum while maintaining the bottle stresses below the glass strength.

5. SPENT FUEL FLASK

After leaving the reactor, nuclear fuel is sent to the spent fuel pool. Some time before the latter is filled the fuel must be transported, either to a reprocessing plant or to a storage facility, be that temporary or permanent. All these activities require the use of various types of storage and transport casks.

A spent fuel cask must fulfil a number of conditions. From a mechanical point of view, the design requirements include a satisfactory performance during and after a series of postulated accidents. This performance refers, among other considerations, to preserving the structural integrity of the cask, maintaining the isolation of its contents and ensuring that the fuel does not sustain unacceptable damage.

In the present Section, an example is presented of a cylindrical cask undergoing a postulated storage accident. The cask falls unprotected on a rigid plane from a height of 37cm (15in). The impact occurs on the edge of its base and with the centre of gravity of the cask directly above the impact point.

The cask is primarily made of stainless steel, although it also contains lead as gamma ray shielding. The cask is about 5.0m long, 2.2m in diameter and has an overall weight slightly in excess of 1MN. Two bolted lids are used to close the cask. For reasons of space, it is obviously impossible to provide here details of the structure of the cask, fuel basket, etc.

Two codes were used in the dynamic analyses of the cask: DYNA3D [1] and ABAQUS/Explicit [2]. The plots presented here correspond to the DYNA3D calculations.

Fig. 5 presents the mesh used to model the cask, the two lids and the bolts. The mass of the contents was smeared along the inner surface of the cask. The model, which has 4116 elements and 5385 nodes, includes frictional contact surfaces between the lids and the walls of the cask. A 0.1 friction coefficient was assumed to be adequate for those contacts.

Elastic plastic models with hardening were generally used for representing the materials involved (various types of stainless steel, lead, neutron shielding, etc.). The impact calculations were preceded by thermal analyses in order to provide the existing temperature distribution at the time of the impact.

The impact duration is long, again due to the negligibly small initial contact area; it lasts about 23msec. Two vertical velocity histories are shown in Fig. 6, corresponding to the lowest and highest points in the cask. The gradual deceleration process of the top of the cask is evident in this figure, eventually leading to its rebound when the lowest node acquires a positive velocity. Fig. 7 presents two close-up views of the
contours of accumulated plastic strain which, as can be seen, may approach 25% very locally at the impacted edge. No problems are caused in relation to the cask integrity, since about half of the wall thickness remains below the 0.2% strain level even at the impact location.

Fig. 5 Mesh used to model the complete cask with bolts and lids

Fig. 6 Velocity histories of the lowest (A) and highest (B) nodes in the mesh
Fig. 7 Two close-up views of plastic strain contours (A=0.002, B=0.05, C=0.10, D=0.15, E=0.20, F=0.25)

While this type of corner impacts are rather demanding on the cask structure, the corner crushing acts as a shock-absorbing mechanism. Thus, peak rigid body decelerations for the cask are only about 25g and the bolts give no reason for concern in this particular impact.
Overall, slightly over 20,000 integration time steps were required for the analysis of this problem, consuming about 3 CPU hours in a Convex C-220.

6. RIGID MISSILE PENETRATION INTO CONCRETE

Various types of accidents, such as the failure of a turbine rotor, are able to produce highly energetic, relatively rigid missiles. These missiles then interact with structures and it is of obvious interest to evaluate the potential consequences.

Impacts by rigid missiles, when they result in considerable penetration of the target, are particularly difficult from a numerical standpoint. This is primarily due to the very large deformations caused to the target in the area directly affected by the projectile. These very large deformations eventually lead to tangling of the mesh, negative volumes of elements and, in short, the impossibility to pursue the calculations. To deal with this problem, the main techniques currently available are rezoning and erosion algorithms. In the first case, new regularized meshes are constructed with varying degrees of participation from the analyst; then, all variables are mapped from the old to the new mesh and the calculations proceed based on the new mesh until rezoning is again necessary. In the second case, elements which have distorted beyond some user-specified limit are simply removed from the mesh. Each procedure has a number of advantages and at least as many disadvantages. The calculations presented here employ a rezoning algorithm.

The problem considered is the normal penetration by an inert military projectile on a reinforced concrete target [4]. The projectile has a mass of 547 kg, length of 2.0 m and diameter of 0.4 m; the nose is ogive shaped and the impact takes place at a velocity of 250 m/sec. The reinforced concrete target has a thickness of 2.0 m and an unconfined compressive strength of 45 MPa.

Since the problem has axial symmetry, only a two-dimensional analysis is required in this particular case. The program selected for the calculations was DYN42D [5].

The mesh used to simulate the problem is shown in Fig. 8. Solid elements have been used to represent all materials, including the inert explosive inside the projectile. Only one element across the steel shell is necessary, since it is known from the experiments that the deformations in the steel are very small and without any tendency to initiate buckling or other instabilities. The mesh shown in the figure actually has twice the number of elements of that used in the calculations since a reflection has been added to improve the visualization. The real mesh is composed of 556 nodes and 483 quadrilateral elements.

Although a number of different analyses were conducted using different constitutive assumptions for the reinforced concrete, only one is presented here for brevity. The concrete model provided in DYN42D has been used together with a mixture rule for deriving stiffness and strength properties for the composite concrete-reinforcement material. A 7% reinforcement was assumed to be present in the outer 16 cm of the concrete target.

As mentioned earlier, it is necessary to rezone the mesh from time to time because of its large distortions and their consequences on both accuracy and the size of a stable time step. Two rezones of the mesh were typically
sufficient in each of the analyses conducted.

Fig. 8 Mesh used to model the projectile and the target

The global evolution of the problem can be followed by inspection of Fig. 9, which presents the histories of vertical velocities at four points, two

Fig. 9 Velocity histories of two points in the projectile (A, B) and two in the target (C, D)
in the projectile and two in the target. As can be seen, the projectile deceleration is almost linear, which indicates that the force developed in the penetration process does not change very much with the depth of penetration. The history culminates with a slight rebound of the projectile, after about 1m penetration, some 7.4msec after the initial contact was developed. These numbers compare well with the experimental observations.

Additional details are shown in Figs. 10 and 11, where contours have been plotted for plastic strains and pressures, respectively, after 4.8msec; at this time, the penetration process is still underway. Fig. 10 clearly indicates the narrowness of the large strain region caused by the penetration. The typical bulb of high pressures ahead of the projectile is depicted in Fig. 11.

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**Fig. 10** Contours of equal plastic strain ($A=0.5$, $B=1.0$, $C=1.5$, $D=2.0$)

**Fig. 11** Contours of equal pressure ($A=-200\text{MPa}$, $B=-100\text{MPa}$, $C=0\text{MPa}$, $D=100\text{MPa}$, $E=200\text{MPa}$)
The number of integration time steps needed to span the duration of the problem was in this case close to 15,000. However, being only a two-dimensional problem, about 8 CPU minutes of Convex C-220 were sufficient to complete the calculations.

7. CONCLUSIONS

In the present paper, several aspects of impact problems have been discussed. The speed, flexibility and simplicity of Lagrangian methods with explicit integration make them the obvious choice for numerical simulation of such phenomena.

Three different problems have been mentioned, with impact velocities ranging from a few to hundreds of metres per second. Also, a variety of computer programs has been used (DYNA2D, DYNA3D and ABAQUS/Explicit), albeit all of them within the explicit Lagrangian family.

The results were in all cases very satisfactory, both in the performance of the programs and in the capabilities of the Convex C-220 computer in which the analyses were conducted. With the exception of the contact logic, for which it is difficult to reap benefits from vector/parallel architectures, the rest of the operations involved in impact problems are implemented naturally and efficiently in this type of architectures.

8. REFERENCES


