REVIEW OF CONSTITUTIVE MODELS TO PREDICT PLAIN AND
REINFORCED CONCRETE BEHAVIOUR UNDER SEVERE LOADS

F. MARTINEZ, Principia S.A.
J. MARTI, Principia S.A.
M.P. NIELSEN, ÍTH-DIC
V.M. TRBOJEVIC, Technica Ltd
A.G. YOUNG, University of Leicester

Summary
This paper contains a review of constitutive models to predict the
behaviour of reinforced concrete under severe loads. A survey of
constitutive models of mass concrete, the behaviour of concrete steel bond
and its numerical implementation are described herein.

1. INTRODUCTION

This paper contains a survey of constitutive behaviour and analytical
techniques required for predicting the structural response of concrete reactor
containment buildings to severe accident conditions. It combines the results of
three projects [1], [2], [3] carried out as Shared Cost Actions within the Reactor
Safety Program of the Commission for the European Communities.
The containment building is the last physical barrier against
uncontrolled releases of radioactivity to the environment. The integrity of the
containment ensures that the fission products will be retained in the event of
design basis accidents but, in addition, it mitigates the consequences of accidents
beyond the design basis.

It is only by carrying out numerical simulations of the overall structural
response up to collapse that the survivability of the system can be assessed.
Therefore, the existence of validated analytical models which is a prerequisite for
any numerical analysis, is of the greatest importance.

Because the validity of inelastic analyses is often limited by the inadequacy
of the material models adopted, the Joint Research Centre at Ispra has given a
high priority to the review of existing constitutive models for concrete. Such
models must be able to describe concrete, with and without steel reinforcement,
across the complete stress range, from initial elastic behaviour to ultimate failure.

This review of mass concrete is divided in three parts, dealing
respectively with constitutive behaviour, modelling of the concrete-steel bond and
numerical implementation for analysis of practical problems.

It is generally accepted that reliable modelling of reinforced concrete
under extreme conditions requires the use of the so-called segmental models.
These models describe independently the response of concrete and steel with
some link conditions. This is the underlying reason for dedicating independent
sections of this review to the constitutive modelling of mass concrete and to
the behaviour of the concrete-steel bond. No attempt has been made to review the
less promising combined models, which incorporate both concrete and
reinforcement response in a single equivalent material.

The study of mass concrete models has been focused on the features
necessary to describe the near-peak and post-peak stages of deformation in a
consistent way, i.e. objective with respect to the mesh. Special attention has
been given to the tensile behaviour of concrete and its softening response after
reaching its tensile strength. Time-independent crack and band models are
described, as well as the latest developments of continuum damage mechanics.

An adequate bond model is necessary in order to represent correctly the
stress transfer between concrete and steel. Thus, in the second part of the paper, a review of the different models for representing the concrete-steel bond has also been conducted, including its behaviour under monotonic, sustained, impact and cyclic loading.

The third part of the paper considers the numerical implementation of the models in computer codes, as well as the existing experimental validations of the codes. Different modelling options for reinforced concrete, bond elements and cracking procedures are described.

2. CONSTITUTIVE MODELLING OF MASS CONCRETE

It is essential to note that the ability of models to describe consistently the near-peak and post-peak stages of concrete loading is rare, and requires very special formulations. Approaches based on stress-strain relationships alone generally lead to inconsistent results due to softening and strain localization.

Softening is a loss of load - carrying capacity when the strain increases. It has been used often in describing concrete failure, although in the early models only abrupt (brittle) softening was considered, so that when a certain criterion was met the strength suddenly dropped to zero.

Today, many models incorporate progressive softening instead of abrupt softening. However, the problems they pose are essentially the same.

The use of a softening stress-strain relationship frequently leads to mathematical results inconsistent with experiments and introduces inconsistencies in the numerical approximations (see [4]). In particular, for time-independent softening models, fracture can happen with no energy dissipation because of the possibility of a strain-localization in a zone of vanishing volume. Indeed, the zero dissipation mode is the thermodynamically preferred mode. In discredited numerical approaches, such as the Finite Element Method (FEM), the minimum width of the localization zone is limited by the size of the elements and the dissipated energy becomes proportional to the element size. Hence, the results are, in general, mesh-unobjective, in the sense that they are mesh-dependent and do not converge to reasonable results when the mesh is indefinitely refined.

A considerable research effort has been devoted in recent years to the tensile failure of concrete. Reinhardt [5] has given a number of reasons for this development, but the essential underlying cause is the need of general purpose codes for numerical structural analysis.

Tensile behaviour is of primary importance for some kinds of structures and loadings, such as bending of beams, punching of slabs and shells, anchors, and shrinkage and thermal cracking. Since a general purpose code must give adequate results to these particular problems, the tensile behaviour must be included as an integral part of numerical codes.

The results have thrown much light on the problem of softening and strain localization and have allowed the bridging between stress-strain approaches and classical fracture mechanics.

When subjected to tension, concrete first deforms under increasing load, then the load reaches a maximum and further deformation takes place with decreasing load. This last stage is the softening regime. During the pre-peak stage, the strain is nearly homogeneous; but at some point around the peak, strain localization starts. In the final stages, the localization appears as a discontinuity (crack), the specimen breaks in two pieces and complete softening is achieved.

As already stated, the use of a softening stress-strain relationship leads to inconsistent, mesh-sensitive numerical results. These drawbacks are overcome - within the frame of classical, local theory of Continuum Mechanics - by postulating some "localization criterion" or, according to Bazant [6], localization limiters. While Bazant interprets the term "localization limiter" as a mathematical restriction forcing the strain-softening region to have a certain
minimum finite size, one may extend this concept to situations where mathematical restrictions force the dissipated energy within the strain-softening region to attain a non-vanishing value. This latter interpretation is useful for classification purposes because it enables apparently different models such as Bazant's Crack Band Model and Hillerborg's Fictitious Crack Model to be included in a common conceptual framework, although at the expense of accepting a different mathematical treatment for the hardening and softening regimes. For fictitious crack models, stress-strain variables must be made on hardening and stress-displacement variables on softening; for band models, stress-strain variables are used on hardening and softening.

It is well known that concrete also displays softening in compressive states of stress. Considerable efforts have been made in the past to include such behaviour in the development of stress-strain relations, but the problem of consistent handling of softening in compression is still open. This is so because strain softening is related to localized modes of failures and strain localization is more difficult to handle in compression than in tension. In tensile states, experience shows that cracks form perpendicular to the maximum stress direction, so that if stresses do not rotate, the bulk and crack deformations are coaxial. This does not happen in compression, where the strain localization takes the form of narrow bands or cracks inclined with respect to the principal directions of imposed stress and deforming in shear mode [7, 8].

In tension, consistent methods have been developed to describe the softening behaviour as a material property; this is achieved by interpreting the displacements in terms of crack openings, rather than in terms of global strains. This is not so for compressive states of stress, although the results of Van Mier [7] point in such a direction, and a method to interpret the experimental results within the frame of band models has recently been put forward by Ortiz [9].

The softening behaviour, described as a stress-strain curve, is unfortunately very sensitive to specimen size and boundary conditions.

2.1 Classical Approaches to Inelasticity and Fracture of Concrete

2.1.1 Elastic-Brittle Models

Elastic-Brittle models correspond to the early stage of nonlinear modelling of concrete behaviour. Such models were developed and used as a first rough approximation during the early seventies. At present, they have been completely superseded by other more sophisticated models.

The essential hypothesis of Elastic-Brittle models is that the behaviour of the material at a point is linear-elastic (and usually isotropic) until a certain criterion is met, at which moment the adequate stress components, as well as the corresponding stiffness, drop suddenly to zero [10].

Two essential forms of failure are considered: tensile cracking and crushing. For tensile cracking, it is assumed that cracks develop suddenly along planes perpendicular to the maximum principal stress direction. In this way, the stress component normal to the crack drops to zero and remains zero for as long as the crack remains open, but the material is still able to transfer stresses in directions parallel to the crack plane.

Detailed analyses of particular situations may be found in Chen's book [10] and references therein. For crushing, it is assumed that, as soon as the fracture criterion is met, all stress components drop suddenly and permanently to zero.

2.1.2 Elastic-Plastic-Brittle Models

The next step in modelling concrete mechanical behaviour was to introduce the concepts of the classical theory of Plasticity. Elastic-Perfectly Plastic models and even Rigid-Plastic models have been used to analyze structural
behaviour (see [10]) However, such models could not describe the nonlinearity of concrete even for the simplest stress paths, not to mention its fracture behaviour.

The obvious way to handle the problem was to use strain-hardening plasticity and a sudden stress-drop when a certain was met. Tensile-cracking and crushing may be separately considered following the same procedures as for Elastic-Brittle models. Unfortunately, this approximation suffers from the same drawback as the Elastic-Brittle models; the numerical results are mesh-unobjective. Moreover, a general algorithm of the kind found by Bazant and Cedolin [11] for Elastic-Brittle models is not available to make the results mesh-objective.

2.1.3 Non-Linear Elastic, Total Strain, and Rate-Type Models

Non-Linear Elasticity has been used to describe the stress-strain behaviour of concrete. As is well known, elasticity implies reversibility, while concrete exhibits highly irreversible behaviour. However, when monotonic proportional loading is considered, the material does not display its irreversible nature and the mathematical tools developed for elastic theories are helpful in describing the observed nonlinear behaviour.

Hence, even though the mathematical equations used are of Non-Linear Elastic type, one has always to have in mind that in reality the behaviour is plastic-like, and that such formulations are strictly valid only when the loading is monotonic and proportional. Taking this into account, the term Elastic, whilst used in practice [10], appears to be somewhat misleading and Total-Strain Theory, as defined by Bazant, may be a clearer denomination.

Total-Strain Models are path-independent by their very definition, in which an algebraic relationship between total strains and stresses is postulated to exist. Concrete behaviour is known to be path-dependent even for continuous loading (no unloading, in the sense of the Theory of Plasticity). Hence, the Total Strain Models are to be used with care when highly non-proportional loadings are considered. In order to relieve the path independence of the Total strain theory, the rate-type models have been proposed.

2.2 Crack Models

Special versions of the theory of cohesive or fictitious cracks, arising from different physical motivations, have been in use for quite a some time. Barenblatt applied this concept by considering attractive atomic forces in a small region near the crack tip [12]. Dugdale proposed a mathematically similar, but conceptually different, model with finite crack tip stress [13]. A new interpretation of these concepts was made by Rice, who assumed for elastic materials a restraining stress as a function of the separation distance [14]. The application of the underlying concepts to concrete fracture was pioneered by Hillerborg and coworkers [15]. Since then, a good deal of theoretical as well as applied work has been performed in which cohesive or fictitious crack models play a paramount role.

It is worth noting, however, that despite the efforts made in the theoretical and experimental fields, a fully general theory of cohesive crack development is still lacking. The theory is well established for isotropic solids, for pure opening (Mode 1), and for monotonic loading (continuously increasing crack opening), but the cohesive crack behaviour for anisotropic media, mixed modes, or non-monotonic opening cases, is still subject to conjecture.

Models based on Linear Elastic Fracture Mechanics appear frequently in concrete fracture literature. Such models use concepts like stress intensity factors, R curves and J integrals, especially suited for small scale yielding problems. It is not advantageous to consider these models as a separate family from the above mentioned, not only because unity and simplicity are lost, but
mainly because they contain ingredients from Progressive Fracture Models.

Pure cohesive crack models do not allow a stress singularity to occur at the fictitious crack tip. Recently, models have been proposed by Shah and co-workers [16] where the singularity at the cohesive crack tip is permitted while retaining the remaining features of the cohesive crack model; in particular, the stress-transferring capacity of the newly formed crack.

The only essential change in the hypothesis previously stated for cohesive crack models regards the crack formation condition. In these models, new cohesive crack surfaces can be created only from a pre-existing crack tip and, consequently, these models are suited for crack growth analysis rather than for a description of the crack nucleation processes.

For pure opening mode, a pre-existing crack will extend when the stress intensity factor at the crack tip reaches a critical value for crack formation; but the newly formed crack surfaces are stress-transferring and the transferred stress will be given, in the simplest formulation, by a function of the crack opening.

2.3 Band Models

At first glance, band models could be considered as a formalization of the smeared crack approach. But the smeared crack concept in itself was initially introduced as an approximate expedient to handle cracking in the frame of numerical methods rather than as a consistent approach to model fracture behaviour. In this approach, used essentially for concrete and rock, the displacement jump due to a crack was averaged over the whole finite element, the advantage of leaving untouched the mesh topology instead of splitting the finite element in two, which had. In the simplest form, the stiffness matrix of the element was set to zero after the cracking occurrence, as dictated by a the cracking criterion.

In the late seventies, a systematic analysis by Bazant and Cedolin showed that this approach was mesh-unobjective and that an energy preserving algorithm should be used instead of a fixed cracking criterion.

This led to mesh-objective results consistent with LEFM concepts, but at the expenses of using a mesh-dependent cracking criterion, which was not conceptually satisfying.

It was only in the early eighties, mostly due to the work of Bazant and co-workers, that the band approach took shape as a theory advocated as independent of the numerical method used to solve a particular problem.

However, except for the simplest loading cases and models, it is very difficult to avoid linking the band model to the numerical algorithm used to solve particular problems.

3. BEHAVIOUR OF CONCRETE-STEEL BOND

It is possible to identify at least three different scales of phenomena which are of interest when studying the interaction between concrete and the reinforcement. At the atomic level, there exists the chemical adhesion between both materials, which is broken when a given shear or tensile stress at the interface is exceeded. After this stage, relative displacements between concrete and smooth bars become possible and a friction mechanism, due to small-scale oscillations of the contact surface, may become active. Thirdly, if the reinforcing element has indentations or ribs, then an interlocking mechanism, which forces the concrete and the steel to move together at some locations of the interface, is mainly responsible for the bond forces.

It is well recognized in the literature that the bond between reinforcement and concrete involves complex phenomena and is affected by many parameters. Research on bond has been mainly directed to the experimental determination of the influence of different parameters. Some analytical models
have been developed, but they usually have a limited scope of application. Some of them make use of well known solutions of linear elasticity or plasticity to determine the cracking pattern of the concrete surrounding the bar [17]. Other approaches assume that the cracked concrete adjacent to the bar behaves as a rod-like structural element and transfers stresses between the reinforcement and the concrete mass [18], [19]. There are also some models which take advantage of knowledge on concrete fracture mechanics to predict the cracking pattern in the vicinity of the rods [20], [21], [22], [23], [24].

Clearly, in connection with the behaviour of concrete containments subject to conditions of severe loading, the models required must be valid under generalized states of loading and must also include other effects, such as temperature and ageing. Such general models can be proposed in the present state of the knowledge only in an approximate fashion as curve-fits of experimental measurements.

Some basic aspects of analytical approaches need further validation. For instance, recent experimental work suggests that the assumption of a unique bond stress-slip relationship, as usually employed in numerical analysis, may not be adequate, since sometimes bond stresses vary in practice in seemingly unpredictable manners [25]. If this were confirmed, a radically different analytical approach might be required [26]. It has been found that experimental measurements of the bond stress-slip relationship vary from one location of the interface to another [27].

This is further evidence that a one-to-one bond stress-slip relationship may not exist for the parameters currently thought to be significant. The main reason for such variability are the many factors influencing the bond: the strength of concrete, mainly the tensile strength; transverse compressive forces, confining reinforcement and concrete cover; geometry of the reinforcing bar; position of bars during casting; environmental effects such as moisture content in concrete, temperature; rate and cycling effects.

3.1 Bond Models

3.1.1 Bond Behaviour under Monotonic Loading

Most of the empirical information available on bond behaviour refers to experiments performed under monotonically increasing load. As a result of such tests, it appears that the bond characteristics of reinforcing bars are best described by means of a bond stress-slip relationship [28]. This approach greatly simplifies the problem of developing models; it implies that complex phenomena, such as concrete cracking, crushing, friction, etc., are all implicitly accounted for through the parameters of the bond stress-slip relationship.

As suggested, earlier, a typical bond stress-slip relationship begins, at the lower stress levels, with a region where chemical adhesion is predominant; in this region the bond stress increases with very little relative displacement [28], [29]. When a given value of the stress is reached, chemical adhesion breaks down and a mechanical type of bonding, which depends upon the surface characteristics of the bar, becomes active.

The subsequent behaviour of the bond will be mainly influenced by the stress induced in the concrete. Further loading in plain bars will frequently cause the shear strength of concrete to be exceeded at relative low values of the applied load, thus causing bond failure by pull-out of the bar [28].

In the case of ribbed bars, the stress state will generally be different, as important tensile and compressive stresses are induced in the concrete mass as a consequence of the ribs. In this manner, internal conical cracks are generated near the bar surface, starting in front of the ribs [30], [31]. This generates a large decrease in the stiffness of the concrete surrounding the bar, i.e. larger slip increments correspond to successive stress increments [28]. Radial forces
transmitted by the ribs to the concrete mass will act as an internal pressure on a concrete cylinder, thus inducing tensile stresses in the concrete mass [17], [32], [33].

3.1.2. Bond modelling under impact loading

Impact loads are very relevant in the context of severe accidents since impacts on the containment may occur as both the cause and the consequence of a severe accident. Empirical information on bond behaviour during impact loads is again not very extensive, but some general conclusions can be offered.

If concrete is well confined, in such a manner that failure of the pull-out type occurs, then the bond strength of deformed bars increases with increasing rate of deformation. Loads greater than those statically acceptable can therefore be obtained under dynamic conditions. However, if the impact or even in the first one if the load is kept beyond a certain time. Bond stiffness is also found to increase with increasing loading rate. Moreover, the dependence of bond stress on strain rate for a given slip, appears to follow a similar trend to that of concrete in compression or tension [34], [35].

It should be noted that the stress distributions associated with impact loads may be very different from those generated in static tests. This is mainly due to inertial effects, but also to rate-dependent behaviour. Thus, results which are valid for static conditions cannot generally be extended to dynamic situations without prior justification. For instance, it has been found that in impact tests there is an increased tendency towards splitting failure compared with the observations from static tests.

3.1.3 Bond modelling under cyclic loading

Although there are only a few experimental studies dealing with bond behaviour subject to cyclic loading, some relatively systematic work on this topic has been completed. The main problem with this type of work is that different types of failure modes may take place within a single set of experimental tests, thus making difficult the interpretation of the results.

The problem of modelling the behaviour of bond under cyclic loading has been a subject of notable interest, especially in connection with seismic problems. Maybe the first model for the cyclic bond response is that due to Morita and Kaku [36]. It considers two different monotonic envelopes for confined and unconfined concrete and also different behaviour when the bar is loaded in tension and compression. The envelopes are defined by two straight lines selected to fit experimental results. It describes fairly well the first loading cycle, but does not predict the observed cyclic deterioration in bond strength and stiffness. As a consequence, the model can only be considered satisfactory for small numbers of cycles with relatively low peakload amplitudes, less than about 80% of the bond strength.

Tassios's model [28] considers a monotonic envelope the same both in tension and compression. If a given slip threshold is exceeded during loading, the values of the bond stress in the reversed direction are reduced to 1/3 of the corresponding ones in the monotonic envelope. In this way the real behaviour of the bond in subsequent loading cycles is greatly simplified, but the cyclic deterioration of bond strength and friction resistance at peak slip is somehow taken into account. When the peak slip is exceeded, the monotonic envelope is regained. Tassios' model represents an improvement with respect to that of Morita and Kaku in that it considers the descending branch of the monotonic envelope and represents bond deterioration due to cycling. However, some inaccuracies remain in the assumption that the monotonic envelope is regained when the maximum slip is exceeded, as well as in the reduction of bond stresses in the reverse direction to 1/3 of the monotonic envelope.

Other models on the same line have been developed [37], [38]. Such models
try to provide a more detailed description of the cycling process by means of a large number of adjustable parameters. The physical meaning of those parameters is unclear and their value must be derived from test results.

3.1.4 Modelling of Thermal effects

The bond response at increased temperatures is obviously of interest in connection with severe accidents, but it has also been the subject of relatively little study. As temperature increases, the bond strength decreases approximately in the same way as the concrete compressive strength. This tendency continues up to temperatures of about 200° to 400°C. For higher temperatures, the bond strength deteriorates faster than the compressive strength of concrete. For temperatures ranging from 600°C to 800°C, the bond strength practically vanishes [39].

Tests have also been performed in which a constant load is maintained while the temperature increases at constant rate. In these experiments, the slip rate increases continuously with time up to a given temperature at which the slip rate greatly accelerates. This latter temperature is found to be some 200° to 400°C lower than the temperature at which the compressive strength of concrete vanishes in an isothermal monotonic test.

The bond strength has also been tested experimentally with temperature cycling prior to the load test. It has been found that, when loading is applied concurrently with the thermal cycle, the residual bond strength was larger at the lower temperatures than at the higher ones. The situation reversed when the temperature increased and gave a maximum different for the maximum temperature considered (250°C) [39].

4.0 NUMERICAL MODELLING OF CONCRETE

It is only by carrying out a numerical simulation of the complete structural response up to collapse that the survivability of the system can be assessed. A summary of the results of various containment analyses by several investigators [1] shows a wide range in predicted ultimate strengths for containments within a given type. More interestingly is the difference found in the predicted pressure response of the same containment by two investigators. The discrepancies can be explained in several ways:

- by the differences in the definition of the failure criteria
- by the differences in material properties
- by the differences in the analysis techniques.

Out of these three main reasons for discrepancies, the last two will be dealt with in more detail here.

The differences in material representation are mainly due to the reinforced concrete modelling. A standard model for concrete under biaxial stress is still evolving. The uncertainty associated with an analytical description of a reinforced concrete segment under biaxial stress is one of the larger uncertainties in the analysis of concrete structures. The behaviour of concrete structures is dominated by complex nonlinear interactions between the steel and the concrete. It is well known that such interactions can have a major effect on global structural characteristics such as stiffness, strength, damping and ductility.

Moreover, steel-concrete interaction can have a primary influence on damage accumulation and failure modes. All these phenomena are difficult to describe analytically.

The effects of cracks on other properties is not usually incorporated into the analysis. Perfect concrete quality is usually assumed. In other words, mapping from the real material to an analytical model is extremely difficult and leads to uncertainties in the test or analysis results.

The differences in the analysis techniques can be based on several factors. Numerical approximation of the physical problem may lead to the popular finite
element or finite difference discretisation. A complete description of the stress-strain state throughout the model is obtained.

Complex material constitutive relationships and geometric nonlinearities are usually included. However, such an analysis has its own uncertainties. Characteristics of the basic finite elements vary from case to case. Solid elements and shell elements incorporate different features. In dynamical situations finite element designers tend to use elements with a lower order of numerical integration than in the static analysis.

Improving the performance of certain elements may lead to suppressing some other types of physical behaviour. The accuracy of computations is affected by element size, aspect ratio, orientation, etc., in addition to the assumptions involved in the element formulation.

Nonlinear problems always involve some kind of iterative procedure. The convergence characteristics of the procedure can affect the accuracy of the solution. The extreme load analysis of concrete structures leads almost inevitably to numerical stability problems.

These problems are usually encountered in tension cracking or softening which causes a change of sign of the solution Jacobian. It is not certain whether these numerical instabilities are associated with physical instabilities.

Different modelling assumptions, e.g. discrete representation of reinforcement, or a "smeared" (orthotropic) model may lead to discrepancies. The definition of the failure criteria for reinforced concrete is still one of the unresolved problems in containment analysis.

It is possible to state that the analysis of reinforced concrete structures involves many uncertainties, some of which have been mentioned in this section. Since each of these uncertainties represents a potential difference of opinion of an analyst or a structural engineer, the following sections will emphasize some of the main topics in the modelling of reinforced concrete structures.

4.1 Nonlinear Solution Strategy

The best known method for solving a large system of nonlinear equations iteratively is the Newton-Raphson method. The Newton-Raphson algorithm can be a very powerful tool for the solution of many nonlinear problems; however it fails for unstable load-displacement relationship.

The unstable load - displacement relationship can be achieved as a solution path beyond limit points. The concrete tension softening can produce a negative tangent stiffness matrix. The usual Newton-Raphson method of load incrementation and iterations within a fixed load increment would break down round the peak, and would not be able to follow the load-displacement path shown.

A significant effort has been made in the last decade towards improving the efficiency, stability, and the range of application of Newton-Raphson type solution schemes in nonlinear structural mechanics.

This effort has been channeled into three areas:

- Optimising of the Jacobian (stiffness matrix) updating, as in the modified Newton-Raphson scheme, or in BFGS scheme [1] Broyden-Fletcher-Goldfrab-Schanno.
- Introducing different constraints to bound iterations within an increment of Newton-Raphson scheme, based on the displacement control and enabling the solution of snap-through or snap-back buckling phenomena.
- Controlling the solution process by automatic load stepping.

The incremental Newton-Raphson schemes suffer from several basic shortcomings. To simplify the discussion the presentation [1] will be followed. The incremental Newton-Raphson algorithm is interpreted as a linearly bound (fixed load step) hyperlinear extrapolation of the solution curve by means of a
full, modified or BFGS updated stiffness. The succession of extrapolation (iterations) is bound by the intersection with a hyperplane defined by the prescribed load level. For such a process, it may be difficult to guarantee the intersections especially for structural stability or softening problems for which a Jacobian indefiniteness and change of sign may occur.

To solve such problems, the surface bounding the Newton-Raphson extrapolation can be interpreted to be oblique to \( R = \text{const} \). Where \( R \) is the residual force or alternatively, a different function. The process interpretation in such a way naturally leads to different approaches. The first approach proposed by Riks and Wempner is based on oblique linear constraint surface. Crisfield introduced a method called arc length method based on circular constraint surface. Padovan et al experimented with elliptic and hyperbolic constraint surface.

All these schemes, in spite of presenting elegant and logical approaches to solving nonlinear problems suffer from the following shortcomings:

- All displacement control schemes (unlike original Newton-Raphson load control scheme \( R = \text{const} \)) have intermediate solutions, i.e. intersections with the incremental Newton-Raphson extrapolations a priori unknown, which makes calculation for a prescribed target load difficult.
- Defining the shape, obliqueness, arc length, radii, etc. of the constraint surfaces can be a puzzling problem. Oversized surfaces may initiate a nonintersection (non-convergence) in hyperspace and therefore solution failure, while an over tightening constraint surface results in an inefficient and computationally expensive iteration process.
- Problems requiring mixed control, i.e. combination of prescribed displacements and some target loads may be difficult to handle.

There is definitely a need to improve on the solution schemes and the requirements of the improved algorithms are summarized in [1] as follows:

- allow full use of a given load step
- enable convergence to specific load and/or deflection state
- automatic sizing of constraint surface
- possibility to reshape and reorient constraint surface in order to improve convergence characteristics.

Padovan introduces several modifications into the constraint methodology in order to develop improved schemes. For instance, keeping the geometry of the hypersurface unique, basic coordinate transformation can enable flexible reshaping and reorientation. The transformation considered in are translations, rotations and coordinate stretches.

4.2 Reinforced Concrete Modelling

In finite element modelling of a reinforced concrete structures or components there are at least three possible options for the representation of the reinforcement, as follows:

- distributed model
- embedded model
- discrete representation

For a distributed representation the steel is assumed to be distributed over the concrete element with a particular orientation angle.

The constitutive relation used in such a case is based on the composite concrete-reinforcement properties and the assumption of perfect bond between the concrete and the steel.

In the embedded representation the reinforcing bar is considered as an axial
member built into the concrete element. In three dimensions a complete reinforcement layer can be smeared into the equivalent membrane elements.

The reinforcement elements should be based on the same interpolation function as surrounding concrete elements especially when bond-slip effects are not considered. It is not necessary to introduce additional nodal points in the reinforcement bars and layers, and the compatibility can be satisfied by transformation of the reinforcement nodal displacement to the adjacent concrete nodes.

The discrete modelling of reinforcement has been widely used. Axial bar elements may be used to connect the corresponding nodes of a concrete element. Beam elements can also be used in order to resist shear and bending. The advantage of this type of modelling is that it can account for possible relative displacement between the concrete and steel. This can be modelled by the means of discrete springs between the steel and concrete.

4.3 Bond Modelling

There appear to be several types of bond elements which can account for steel-concrete interaction, as follows.

Bond link element developed by Ngo and Scordelis can be used with the discrete modelling of steel and concrete by means of inserting a spring-based link element without physical dimension. The steel and concrete nodes have the same coordinates. The link is assumed to consist of two springs, one parallel and the other normal, to the longitudinal axis of the reinforcement bar. The relationship between local bond shear stress and bond slip is represented by the longitudinal spring. The spring acting normally to the reinforcement axis transmits normal forces between the steel and concrete and is important in modelling the dowel contribution of reinforcement in resisting shear.

Contact element representation can also be used. Bond is modelled by a four node contact element with a linear displacement formulation. Two nodes of a concrete element are connected to the reinforcement element by a bond element which has no physical dimension in the transverse direction. The material law relates bond shear stress to the slip in the longitudinal direction, and there are two relative displacements at each node, related to the bond normal stress. This element is designated for the two-dimensional problems. An interesting non-linear bond-slip relationship utilizing this element is presented in [1]. The bond shear stress/bond slip relationship in [1] is nonlinear and is described accordingly. Several examples show that the application of this type of bond modelling yields the results which are in satisfactory agreement with the appropriate test measures. This bond model in ADINA was later extended to quadratic and cubic interpolation functions, describing curved one dimensional elements and also in two dimensions [1]. In this later model the shear carrying capacity of a cracked joint which depends on the magnitude of force normal to the joint was achieved by the coupling of the maximum shear force with the normal force. The cyclic shear force/slip relationship was modelled according to tests on smooth joints.

Bond zone elements belong to the third group and differ from the two types described previously. The main difference is their finite dimension. These elements model not only the contact surface between the steel and concrete, but also the concrete around the reinforcing bar, this forming a bond zone. A constitutive model is implemented accounted for a special properties of this zone.
Mixture element for bond action reviewed in [1], is based on analytical "mixing" of steel and concrete. A variational principle is used to synthesize a binary mixture model of the composite from the constitutive relations of the components and interfaces, and the steel geometry. Steel-concrete interaction in this methodology appears in the form of mixture interaction terms.

4.4 Code Validation

Investigations based on numerical techniques must be necessarily supported by experimental results which prove their reliability. Probably the best way of conducting such validation in relation to the topic of this paper consists in performing numerical studies to try to reproduce small-scale laboratory experiments. Once that achieved, the validated laws can be used to predict at the large-scale structural level.

The possibility of extending simple physical behaviour to study a more complex structure is achieved by means of a mathematical formulation able to describe the macroscopic physical behaviour of solids.

A great effort has been made in recent years to advance on this task of structural validation. Of special relevance here is the pressurization of failure of a scaled Containment Integrity Program sponsored by the U.S. Nuclear Regulatory Commission. The overall objective of the program is to develop test validated methods that can be used to predict the performance of light water reactor containment buildings subject to loads beyond the design basis [1].

High pressure testing of the containment model was completed in July 1987 by pressurising the containment with nitrogen gas until pressure could no longer be maintained in the containment model by the pressure source.

Large amounts of data were generated during the testing of the 1:6 scale reinforced concrete containment. The observations made during testing have helped to focus analytical efforts on the features which govern containment performance.

Pre-test analyses of the 1:6 reinforced concrete containment model have been conducted by a number of organizations in the United States and Europe [1] in order to predict the response and failure of the model, caused by the static internal pressurisation. Results and comments of this pre-test analysis are summarized in [1].

There are currently some new attempts at validation in the area of overpressurization. The Central Electricity Generating Board in the United Kingdom is expected to carry out a pressure test to failure on a one tenth scale prestressed concrete model of the Sizewell B containment building. This experiment will be undertaken as a regulatory requirement in order to validate the computer codes used in the design and analysis of the full scale structure [1]. A present, it is still unknown whether the validation of an exercise similar to that of the Sandia experiment will be at all possible. It is hoped that the Central Electricity Board will make public enough data to make feasible a similar set of comparisons with analytical approaches.
For reasons of space, detailed references could not be included here and are listed in [1], [2] and [3].

5. REFERENCES


Bond Interaction Models.
In "Bond and Anchorage of Reinforcement in Concrete", Chalmers University of Technology, Goteborg, Sweden.

Über die Grundlagen des Verbundes zwischen Stahl und Beton.
Deutscher Ausschuss für Stahlbeton, Heft 138.

[31] GOTO, Y. (1971)
Cracks Formed in Concrete Around Deformed Tension Bars.

A Theory of Bond Applied to Overlapped Tensile Reinforcement Splices for Deformed Bars.
Chalmers University of Technology, Division of Concrete Structures, Publication 73:2, Goteborg, Sweden.

[33] ELIGEHAUSEN, R., POPOV, E.P. AND BERTERO, V.V. (1983)
Local Bond Stress-Slip Relationships of Deformed Bars under Generalized Excitations.
Earthquake Engineering Research Center, College of Engineering, University of California, Berkeley, California, Report UUCB/EERC-83/23.

Bond Resistance of Deformed Bars, Plain Bars and Strands under Impact Loading.
Stevin Laboratory, Concrete Structures, Delft University of Technology, Report 5-80-6, Delft, p 84.

Influence of Loading Rate on Bond Behaviour of Reinforcing Steel and Prestressing Strands.
Materiaux et Construction, Vol. 15, No 85.

Local Bond Stress-Slip Relationship under Repeated Loading.

[37] VIWATHANATEPAS, S., POPOV, E.P. AND BERTERO, V.V. (1979)
Effects of Generalized Loadings on Bond of Reinforcing Bars Embedded in Confined Concrete Blocks.
Earthquake Engineering Research Center, Report No UCB/EERC-79/22, College of Engineering, University of California, Berkeley, California.

[38] ELIGEHAUSEN, R., POPOV, E.P. AND BERTERO, V.V. (1982)
Local Bond Stress-Slip Relationships of Deformed Bars under Generalized Excitations.

Response of the Bond in Reinforced Concrete to High Temperatures.