EXPLICIT MODELLING OF IMPACT AND PENETRATION

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INTRODUCTION

Solution procedures for non linear dynamic problems in Solid and Structural Mechanics may be grouped into two categories: explicit integration schemes and implicit integration schemes. The former do not require the assemblage of system matrices (stiffness, damping, mass) since the governing equations can be uncoupled and local approximation suffice. On the other hand, implicit schemes usually need the triangularization of a (changing) global stiffness matrix in order to solve the incremental equations of the complete system. This difference highly conditions code architecture and determines the characteristics and applicability of both methods.

Explicit procedures have been used traditionally for problems dominated by high frequencies such as impact and fast transients, where the limitation on the time-step imposed by their conditional stability is not a factor.

Another advantage of explicit methods we would like to point out is that, in spite of their conditional stability, they possess and inherent robustness when dealing with material, geometric, or structural softening. This is due to the fact that they need not achieve convergence in solution of a global system of equations as implicit methods do.

Hence, although implicit models are preferable generally for slow loading or inertial vibrations, some highly non-linear problems of this kind make attractive the use of explicit procedures.

In this paper we describe first a finite difference model with explicit time integration, and then we present several examples of application including a pipe impact, a tube collapse, an analysis of inversion tubes, and an analysis of the penetration of a projectile into metallic armor. Computer calculations have been made using the vectorized codes PR2D [1], PR3D [10] and ARMI [2] on a Convex C120 minisupercomputer and on a CRAY-1 supercomputer.
FINITE DIFFERENCE MODEL

A semidiscretization approach is employed in which independent discretizations are performed for space and time domains. In space, the mesh is Lagrangian (i.e. it follows the material distortion), and approximations are obtained directly from centred finite difference formulae. The motion is described using the deformed configuration through the Cauchy's stress tensor (\(\sigma\)) and the rate of deformation tensor (\(d\)).

Time integration is done according to a computational cycle which is repeated to advance the problem in time. This cycle can be summarized as follows (figures 1 and 2).

The mass is lumped at the grid-points of the mesh. The forces acting at each grid-point are used to compute its acceleration, which is then integrated to yield its velocity and displacement. Relative displacements of grid-points result in forces being produced by several different mechanisms. Firstly, relative displacements of grid-points in a continuum give rise to strains: the constitutive equations immediately generate the corresponding stresses, which are integrated to yield forces. Second, at the contacts between continua, forces are derived from the relative displacements between the boundaries of the two continua in contact. Finally, some boundary conditions (e.g.: non-reflecting boundaries) also result in added forces. All these forces are added together with the body forces and other applied loads to produce the resultant force acting at each grid-point, thus restarting the computational cycle.

Note that the constitutive laws of continua, as well as the contact laws, are always used directly. All variables on the right-hand side of the equations are known, hence the left-hand side can be readily computed without need for iterations or approximations.

The thermal calculations are entirely similar. Internal heat sources, heat flow across element boundaries and plastic energy dissipation combine to change the local
temperature. The new temperatures are then used to calculate the new gradients, the thermally induced stresses and any temperature-dependent material properties.

Each time that the computational cycle is completed, the time is incremented. A necessary condition for the above scheme to work is that the time increment must satisfy the Courant stability criterion. Mechanically, such criterion simply requires that no information be allowed to travel between two grid-points in less than one time step. A somewhat similar criterion applies to the thermal calculations. If the above condition is fulfilled, each grid-point will behave independently from the surrounding ones for the duration of a time step. This effectively uncouples the equations of motion of each node, which can then be solved separately, without needing to solve a system of simultaneous equations for the complete model.

This approach is particularly suited to run in vector computers since the computational cycle can be easily vectorized. In our experience, vectorization of an explicit code can reduce overall execution times by a factor of around five.

**SEMIDISCRETIZATION IN SPACE**

The mesh is based on constant strain triangles and tetrahedra respectively for 2D and 3D. The velocity gradient tensor at the centroid of a cell is computed through finite differences:

\[
[v_{i,j}] = [VR(i,N)] [XR(N,j)]^{-1}
\]  

(1)

where,

\[ v_i = \text{velocity field} \]
\[ VR(i,N) = \text{relative velocities of each node with respect to} \]
\[ \text{the cell's reference node.} \]
\[ XR(N,j) = \text{relative coordinates of each node with respect to} \]
\[ \text{the cell's reference node.} \]

The rate of deformation and spin tensors are computed from:
$$d_{ij} = (v_{ij} + v_{ji}) / 2$$

$$w_{ij} = (v_{ij} - v_{ji}) / 2$$

It is well known that ordinary constant strain triangles and tetrahedra provide overstiff solutions for elastic-plastic problems [4]. Here a Mixed Discretization concept is used [5]. This means that isotropic strains are referred to larger elements than their deviatoric counterparts. For example, in 2D the basic element is a constant deviatoric strain triangle, each two triangles forming a constant volumetric strain quadrilateral.

In practice Mixed Discretization is implemented by assigning for each of the basic elements in a group the mean of their volumetric strains plus a corrective term to account for possible tangling over of mesh [7]:

$$\Delta \varepsilon_V := \Delta \varepsilon_E = CT + (\Sigma \Delta \varepsilon_V V^c) / \Sigma V^c$$

where,

$\Delta \varepsilon_V = \text{volumetric strain increments.}$

$E$ is the group of cells $C$ that form a Mixed Discretization element (two triangles in each quadrilateral for 2D).

$V^c = \text{volume of a cell.}$

$CT = \text{corrective term.}$

The purpose of this Mixed Discretization is to give the mesh enough flexibility to accurately model plastic flow but, at the same time, to use deviatoric forces in order to make the mesh resistant to the zero energy or "hourglassing" modes in a natural way.

Mixed Discretization performs this task efficiently for impact velocities below 500 m/seg as a thumb rule for metallic materials. However, for higher velocities we have found necessary to introduce a further anti-hourglass artificial viscosity [2] [6]. This is so because deviatoric forces are limited by the yield stress value and, beyond this limit, they can not effectively counteract the "hourglassing" of the mesh. At these velocities high pressures develop and deviatoric strength is not a factor.
CONSTITUTIVE EQUATIONS

Constitutive equations are based on a hypoelastic predictor which is then corrected according to the desired material behaviour (Elastic-plastic, Newtonian, Elastic-Viscoelastic ...)

The hypoelastic law is:

\[ \dot{\sigma} = \lambda \text{tr}(\varepsilon) I + 2G \varepsilon - \alpha (3\lambda + 2G) \dot{\varepsilon} I \]  

(6)

where,

\[ \lambda, G = \text{Lame constants} \]
\[ I = \text{Identity tensor} \]
\[ \varepsilon = \text{Temperature} \]
\[ \alpha = \text{Linear thermal expansion coefficient} \]

Dots refer to time derivative.
Using Jaumann's rate of Cauchy stress \( \dot{\sigma} \), the algorithm of hypoelastic stress update can be written as:

\[ \sigma^{n+1} = \sigma^n + (\dot{\sigma}^{n+1/2} - \sigma^{n+1/2} w - w \sigma^{n+1/2}) \Delta t \]  

(7)

The yield surface is

\[ F(\sigma, Q) = 0 \]  

(8)

where \( Q \) is a set of plastic hardening parameters. If condition (8) is exceeded, stresses are modified so that the point representing the state of stress remains on the yield surface. This is done by means of a radial return to the yield locus [8]:

\[ \sigma^{n+1} = \sigma^{n+1} - (\beta/3) \partial F/\partial \sigma \]  

(9)
where $\beta$ is a scalar factor so that $F(\sigma, Q) = 0$

This radial return implicitly defines an associative plastic flow:

$$d^p = \left[ (\beta/3) / (2G) \right] \partial F / \partial \sigma$$  \hspace{1cm} (10)

**EQUILIBRIUM**

Integrating over a small surface $S^{(k)}$ around each node $K$, and using Momentum Conservation Law:

$$u^{(k)} = \left[ \int_S \sigma n \, dS + M^{(k)} \, f + R^{(k)} \right] / M^{(k)} \hspace{1cm} (11)$$

where,

$\ddot{u}^{(k)} = $ aeceleration of node $K$

$M^{(k)} = $ lumped nodal mass

$f = $ mass forces

$R^{(k)} = $ concentrated forces

**TIME INTEGRATION**

Velocities and displacements are updated using central difference formulae:

$$\ddot{u}^{n+1/2} = \ddot{u}^{n-1/2} + \ddot{u}^n \Delta t \hspace{1cm} (12)$$

$$u^{n+1} = u^n + \dot{u}^{n+1/2} \Delta t \hspace{1cm} (13)$$

Courant stability condition demands:

$$\Delta t < h_{\text{min}} / C_p \hspace{1cm} (14)$$
where,

\[ h_{\text{min}} = \text{minimum dimension of cells} \]
\[ C_p = \text{elastic compressive wave velocity} \]

**CONTACTS**

Interaction forces are computed allowing certain penetration between surfaces and imposing penalty forces that are proportional to this penetration:

- Normal force:  \[ F^N_c = K^N p^N \]  \hspace{1cm} (15)
- Tangential force:  \[ F^S_c = \min (K^S p^S, F^N_c \mu) \]  \hspace{1cm} (16)

where,

\[ K^N, K^S = \text{normal and tangential contact stiffnesses} \]
\[ p^N, p^S = \text{normal and tangential penetration components} \]
\[ \mu = \text{friction coefficient} \]

It should be noted that penetration \( p \) is a vector linked to the Lagrangian mesh and hence requires a corotational formulation in its updating, similar to eqn. (7).

The effects of contacts are included without difficulty in the computational explicit cycle. Penetrations are computed at the same time as strains, and interactions forces are considered as nodal forces in eq. (11). However, the code should be capable of automatically detecting new contacts and the sliding and changes in existing ones. The algorithms to do so in a general way are rather complex [9].
APPLICATION EXAMPLES

To show the power and versatility of the model just described, we present in this section several application examples.

The calculations have been made with the codes PR2D [1], PR3D [10] and ARMI [2] using a CRAY-1 supercomputer and a Convex C120 minisupercomputer.

PIPE IMPACT

The problem contemplated is that of a stationary pipe suddenly impacted at equal distance from its extremes by another pipe moving at 50 m/seg. The axes of both pipes are mutually perpendicular, and the direction of the velocity of the moving pipe is perpendicular to both axes (figure 3).

The stationary pipe has a length of 1000 mm, a diameter of 319 mm, and a thickness of 16 mm. The corresponding values for the moving pipe are 1600 mm, 395.6 mm, and 33.5 mm, respectively.

The calculations were performed using PR3D. The material was considered elastic-plastic with a rupture limit on elongation strains.

Figures 4 to 7 show the time evolution of the impact. In figure 8 the contact force time history is plotted.

AXIAL COLLAPSE OF TUBES

This example is taken from a research program on the application of tube collapse as energy dissipating mechanism during impacts [7]. Here we present the analysis of the collapse of an aluminium tube under axial compression.

The calculations were done with PR2D using axisymmetric elements (fig 9). The material was considered elastic-plastic with isotropic hardening. During the impact
the tube develops three successive folds that exert contact and friction forces. Plastic strains, both in the experiment and in the model, exceed 120 %. Numerical predictions compare very well with experimental results (see fig.10 thru 12).

**INVERSION TUBES**

Another impact energy absorber is the so called "Inver" tube device. Basically this is a process which allows a thin-walled ductile metal tube to be turned inside-out or outside-in. In this example we present an analysis of such a device.

We have a tube, as shown in figure 13, with its flared end clamped and we move upwards the other and with a velocity of 50 m/seg. The tube has an outer diameter of 340 mm, and a thickness of 30 mm.

The analysis has been made using the axisymmetrical elements of PR2D. The material behaviour has been taken as elastic-perfectly plastic.

Results are shown in figures 14 thru 18.

**METALLIC ARMOR**

In this example we present the analysis of the impact between a tungsten projectile at 1290 m/seg and a steel armor.

The projectile has a diameter of 12 mm and is 60 mm long. The thickness of the armor is 30 mm.

The calculations were done with ARMI using axisymmetric elements (fig. 19). Both materials were supposed elastic-plastic with piecewise linear isotropic hardening. Influence of temperature and strain rate on yield stress was considered. The volumetric strength was represented by a polynomal equation of state.
The high velocity of the projectile forced to introduce an "anti-hourglassing" viscosity, and the severe distortions of the mesh were treated using an erosion algorithm.

Several stages of the impact can be seen in figures 20 to 24. Figure 25 shows the evolution of projectile velocity.

CONCLUSIONS

Highly non linear problems with large plastic deformations, such as impact problems, are particularly well suited to be treated by Lagrangian numerical schemes with explicit integration. Such a scheme has been presented in this paper.

Integration algorithms may consume large amounts of computer resources for certain problems due to their conditionally stable nature (i.e. limited time-step). However, they are easily vectorized and their vectorization can reduce execution times by a factor of five. Supercomputers with vector processors are then specially adapted to solve this kind of problems.

REFERENCES


4. NAGTEGAAL, J.C., PARKS, D.M. and RICE, J.R. "On Numerically Accurate


FIGURE 1. Mechanical computational cycle

FORCES

VELOCITY/ DISPLACEMENT EQUATIONS

BOUNDARY CONDITIONS

ACCELERATIONS, VELOCITIES AND DISPLACEMENTS

CONTINUUM CONSTITUTIVE LAWS

STRUCTURAL CONSTITUTIVE LAWS

CONTACT LAWS

APPLIED LOADS
Figure 2. Thermal computational cycle.

- Heat Fluxes
- Neumann Boundary Conditions
- Fourier's Law
- Energy Balance
- Temperatures
- Plastic Energy Dissipation
- Dirichlet Boundary Conditions
FIGURE 4. Pipe impact at $t = 1$ msec.

FIGURE 5. Pipe impact at $t = 2$ msec.
FIGURE 6. Pipe impact at \( t = 3 \text{ msec} \).

FIGURE 7. Pipe impact at \( t = 4 \text{ msec} \).
FIGURE 8. Pipe impact. Contact force
   Time - history

FIGURE 10. Axial collapse. View of crumpled tube.
FIGURE 11. Axial collapse. Section of crumpled tube.
FIGURE 13. 'Inver' tube. Mesh view.
FIGURE 14. 'Inver' tube at t = 5 msec.
FIGURE 16. 'Inver' tube at $t = 15$ msec.
FIGURE 17. 'Inver' tube at $t = 20$ msec.
FIGURE 18. 'Inver' tube. Load - stroke curve.
FIGURE 20. Metallic armor at $t = 12 \mu$sec.
FIGURE 21. Metallic armor at $t = 24\mu\text{sec}$.
FIGURE 22. Metallic armor at $t = 48 \mu\text{sec.}$
FIGURE 23. Metallic armor. Projectile velocity time history.