Numerical Methods for Transient and Coupled Problems

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TRANSIENT THERMO-MECHANICAL ANALYSIS WITH EXPLICIT SCHEMES

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SUMMARY

The numerical simulation of metal forming processes usually involves the solution of a coupled system of mechanical and thermal equations. Extreme geometrical and constitutive non-linearities are included in the system.

An explicit Lagrangian finite-difference procedure for solving these problems has been constructed and embodied in the computer program PR3D. The paper presents the theory and describes its application to a case involving very severe distortions and full thermo-mechanical coupling. The predictions are compared with experimental measurements.

1. INTRODUCTION

The motivation for the work presented here is the need to model forming processes such as extrusion, where large strains and thermo-mechanical coupling are clearly present. Any numerical procedure attempting to simulate such processes must account accurately for the large local strains and rotations (geometrical non-linearities). It must also deal accurately with material behaviour up to those large strains, (constitutive non-linearities). The mechanical equations must be solved simultaneously with the thermal equations, as the two are coupled in both directions. Finally, the procedure selected must be able to cope with localisation effects,

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including physical instabilities. To all those constraints, the necessity of predicting the transient, as opposed to steady-state, response of the material must be added in the present case.

The above requirements imply that a time-marching procedure should be used in conjunction with an incremental formulation referred to an updated Lagrangian frame. The very large geometrical non-linearities are otherwise practically impossible to handle.

The time integration could be explicit or implicit. The former has the advantages of simplicity and generality at the price of imposing a small integration timestep. The latter allows larger timesteps but becomes less advantageous as the non-linearities sharpen or wave propagation effects dominate. Further, convergence may be affected by problem characteristics, and difficulties may arise when dealing with real physical instabilities (buckling, necking, etc.).

Here, an explicit scheme has been selected. The small timestep forced by stability requirements has only moderate consequences. It is obviously of little significance in short duration problems. On the other hand, long problems are often quasi-static. This frequently allows contracting the timescale.

2. THEORY

The solution of the thermo-mechanical equations uses here a central finite-difference discretisation. The Lagrangian mesh is based on constant-strain tetrahedrons. Strictly speaking, a mixed discretisation [1] of space is used. The deviatoric part of the tensors is referred to a mesh of constant-strain tetrahedrons, while their volumetric counterparts relates to a mesh of larger elements, each comprising typically three tetrahedrons. This avoids both hourglassing and artificial stiffening during plastic flow. The problem is advanced in time with an explicit central difference operator.

This combination offers a simple and efficient algorithm [2] and has proved very successful for non-linear mechanical analysis, both dynamic (e.g. [3] - [7]) and quasi-static (e.g. [8] - [10]). The method is extended here to encompass coupled thermo-mechanical processes. Three sources of coupling between thermal and mechanical phenomena are considered:

- thermal expansion
- thermal-dependent material behaviour (e.g. lower stiffness, lower yield, lower hardening index, higher rate sensitivity, etc., as temperature increases)
- heat generation from plastic deformation and friction

As mentioned earlier, a rate-type formulation is employed in which variables are referred to an updated Lagrangian frame. As a function of the velocity and temperature fields, the rate of deformation tensor d_{ij} at a point may be expressed as:

$$d_{ij} = 0.5\ (\dot{u}_{i,j} + \dot{u}_{j,i}) + \beta\dot{T}\delta_{ij} \qquad (1)$$

where $\dot{u}_i$ is the velocity field
β is the coefficient of thermal expansion
T is the temperature
δ_{ij} is the Kronecker delta
commas indicate spatial derivatives
dots indicate material time derivatives

Elastic-plastic constitutive laws are implemented using hypo-elastic predictions and a radial return method. The hypo-elastic prediction takes the form:

$$\overset{\Delta}{\sigma}_{ij} = \lambda d_{kk}\delta_{ij} + 2\mu d_{ij} \qquad (2)$$

where $\overset{\Delta}{\sigma}_{ij}$ is the Jaumann rate of Cauchy stress
λ, μ are the Lame elastic constants

The deviatoric components of the new stresses are then scaled back to the current yield surface, resulting in an associative flow rule. The yield surface site is given by a Von Mises criterion including strain-rate and temperature dependence:

$$Y = Y_o(\bar{\varepsilon}_p)\ (1 + B\ |\hat{\dot{\varepsilon}}_p|^m)\ T^q \qquad (3)$$

where Y_o is the reference quasi-static hardening law

$\hat{\varepsilon}_p = \left[\frac{2}{3}de^p_{ij}de^p_{ij}\right]^{1/2}$ is the effective plastic strain

B, m and q are material constants

As the integration requires a small timestep for stability, the process is incrementally linear, obviating the need for sub-stepping or iterations. Large strains and displacements are accurately recovered in this fashion.

The momentum equations are formulated with lumped masses. Integrated on a small space domain around each grid node, they can be expressed as:

$$\ddot{u}_i = \frac{\int_S \sigma_{ij}n_j ds + Mf_i + F_i}{M} \qquad (4)$$

where M is the mass assigned to each node
S is the external surface around the corresponding region
n_j is the normal at each point of this surface
f_i are body forces per unit mass
F_i are external intensive forces

The operation of the above equations is implemented in the fashion shown in Figure 1.a. Each time the cycle is completed, time increases by one integration timestep. Figure 1.b describes the thermal computational cycle.

An energy balance equation is needed to predict temperature variation. When integrated over a small volume V, that equation takes the form:

$$\dot{T} = \frac{\int_S kT_{,i} n_i ds + \int_V s_{ij} \dot{e}^p_{ij} dv + \rho \dot{Q} V}{\rho C_p V} \qquad (5)$$

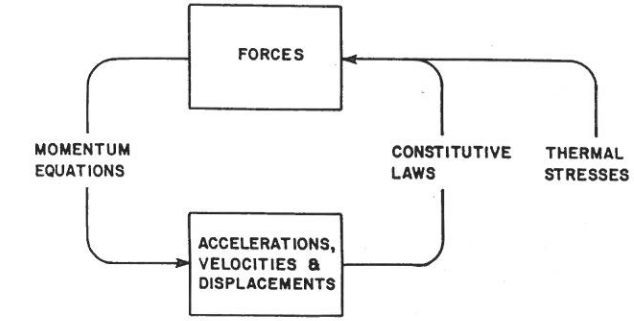
where k are the thermal conductivity
s_{ij} is the deviatoric stress tensor
ρ is the mass density
Q is the heat generated per unit mass
C_p is the specific heat

3. APPLICATION

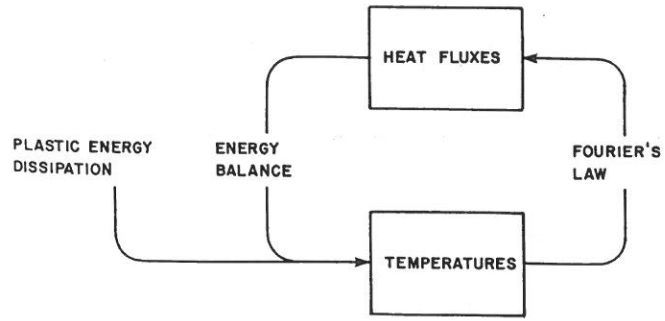
In order to test the formulation, experiments were carried out on 6063 Aluminium alloy. They covered several orders of magnitude of strainrates, as well as temperatures ranging from 25°C to 450°C. Simple as well as complex test configurations were used.

The test configuration of the problem presented here is shown in Figure 2. The 6063 workpiece is initially a right circular cylinder, unstressed, unstrained and at 25°C. The workpiece is compressed axially between a low-friction flat rigid platen and a rough rigid steel block. The block is provided with a square hole through which the alloy can extrude. The distance between the indenting block and platen is decreased by one third (5mm) uniformly over 16msec. The problem is essentially static, as the loading time is much greater than the highest eigenperiod of the system.

The mechanical material properties were determined in calibration experiments. At 25°C, the material was found to behave as an elastic non-ideally plastic solid. Its elastic behaviour was described with a Young's modulus E = 69.7GPa and a Poisson's ratio v = 0.3. A von Mises yield locus was assumed. Hardening was found to be represented by a power-law



(a)



(b)

Fig. 1. Mechanical and Thermal Computational Cycles

with a coefficient of 195Mpa and a hardening index n=2. Due to the type of loading, it is not very relevant whether isotropic or kinematic hardening is used. Since the former is simpler, it was the one selected for representing the material. No rate sensitivity was noticeable.

It is worth mentioning that, at 450°C, the constitutive description was found to be very different. No strain hardening was evident while rate sensitivity became all important. However, the above description could be maintained in the present case as temperatures never exceeded 100°C.

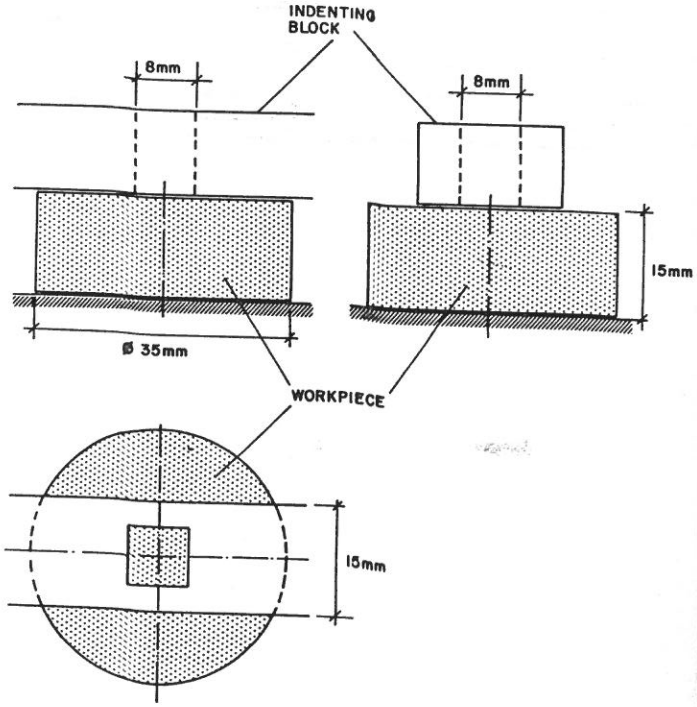


Fig. 2. Test Configuration

For similar reasons, the thermal properties, obtained from the literature, could be considered independent of temperature. The values adopted were a thermal conductivity

$k = 174\ W/m^{\circ}K$, a specific heat $C_p = 900\ J/kg^{\circ}K$ and a coefficient of thermal expansion $\beta = 23.4 \times 10^{-6}\ ^{\circ}K^{-1}$.

As the mechanical problem is independent of the timescale, time can be contracted with two provisos: a) that the contraction is not so large that a non-existent dynamic character is triggered (i.e. inertia forces must remain negligible), b) that the thermal conductivity is suitably scaled. The computational timescale could then be contracted to complete the test in 0.1msec.

4. RESULTS AND DISCUSSION

The two symmetries allow restricting the model to one quarter of the workpiece. The mesh used, shown in Figure 3, consists of 6072 tetrahedrons and 1355 nodes. Each hexahedron in the figure is actually decomposed into six tetrahedrons. The shaded area corresponds to the contact with the block.

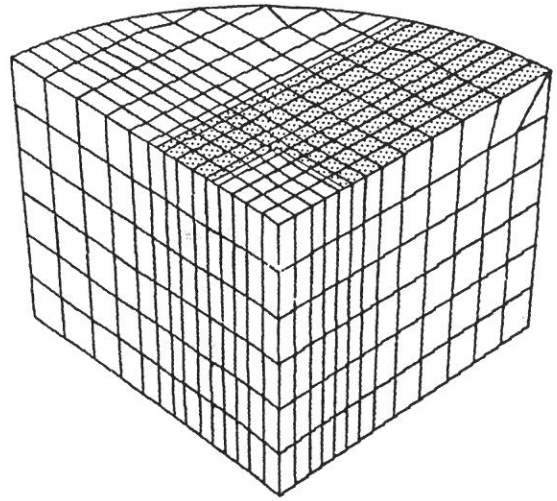


Fig. 3. Numerical Discretisation

Contours of equal effective stress have been drawn on the deformed mesh after an indentation of 5mm (Figure 4). Contour 1 corresponds to 80MPa, from which they proceed in increments of 40MPa. This distribution was confirmed experimentally by microhardness testing on sectioned workpieces.

Temperature predictions are not shown for lack of space. These were compared with the trace of a thermocouple embedded in the workpiece with reasonably good agreement.

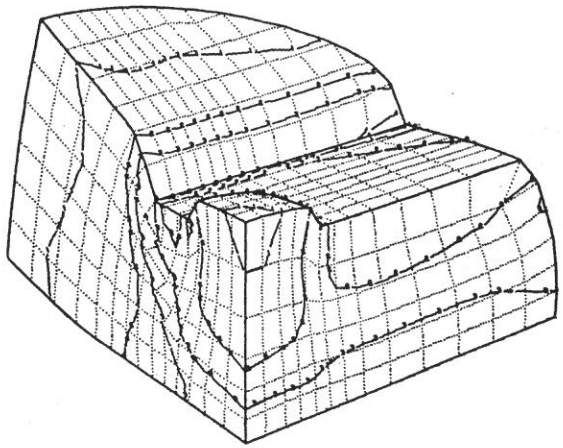
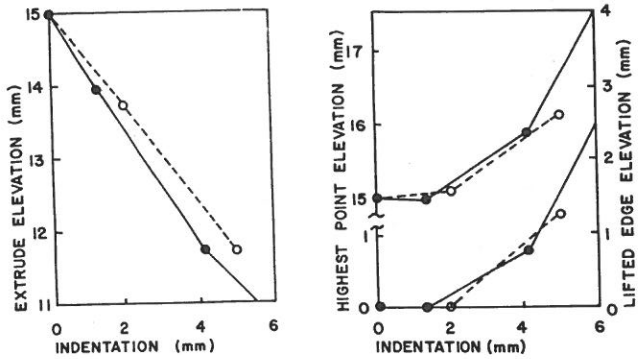


Fig. 4. Effective Stresses at 5mm Indentation

Of greater interest are the predictions of geometrical changes. Under pressure from the indentor, the workpiece curls up losing contact with the bottom platen over part of its initial area of contact. Figure 5 presents comparisons of various geometrical dimensions: a) the elevation of the top point in the workpiece and the elevation of the point on the bottom surface which separates most from the bottom platen; b) the elevation of the top of the extrude; c) the bottom diameter normal to the loading block; and d) the bottom diameter parallel to the loading block. As can be seen, all predictions are excellent except for that concerning the evolution of the bottom diameter parallel to the loading block. This is a consequence of the moderate misrepresentation involved in assuming a frictionless contact with the bottom platen.

The most interesting lesson is learned from the evolution of the indentation loads, paradoxically the worst of the predictions made (Figure 6). The indentation load is clearly overpredicted by the calculations. This overprediction is attributed to the relative coarseness of the mesh, particularly in the region between the extrude and the rising sides of the workpiece.

In a problem described essentially by displacement (or force) boundary conditions, a crude mesh may result in acceptable displacement (or force) predictions. However, the consequences will be noticed when comparing force (or displacement) predictions and measurements.



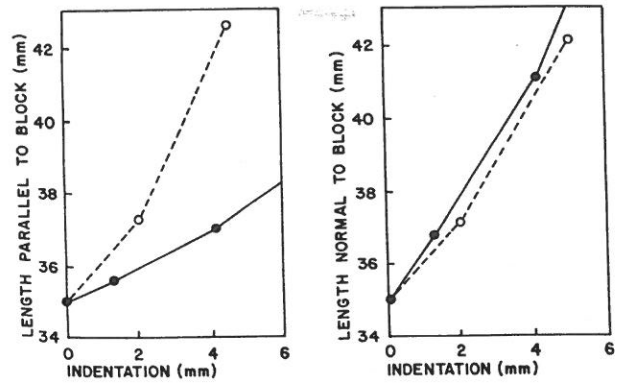


Fig. 5. Comparison of Geometrical Predictions

Similar calculations at 450°C resulted in much better predictions of the indentation load. The reason is that, at the higher temperatures, the material does not harden. Hence, crude meshes can result in reasonably accurate predictions [1]. As the tangent stiffness increases, progressively more refined meshes are needed to maintain the same accuracy.

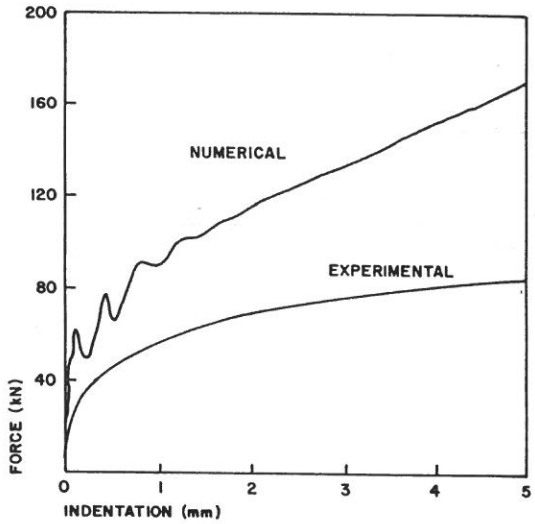


Fig. 6. Comparison of the Force of Indentation

Because of the same absence of hardening, deformations are more localised and extrusion enhanced at the higher temperatures. As a consequence, the problem cannot be handled without incorporating procedures to rezone the Lagrangian mesh. Such procedures are complex and constitute the object of a separate publication [10].

5. CONCLUSIONS

The equations governing the thermal and mechanical behaviour of solids undergoing large three-dimensional deformations have been embodied in the computer program PR3D.

Validation tests result in reasonably good comparisons of predictions and measurements, thus indicating that the numerical procedures and material behaviour implemented do indeed describe well the behaviour of the deforming solid. However, special attention must be paid to the refinement of the discretisation, particularly with hardening materials.

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