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THREE-DIMENSIONAL ANALYSIS OF PIPE-ON-PIPE
IMPACT AND FRACTURE

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SUMMARY

When designing pressurised pipe systems for nuclear installations, it is necessary to consider the failure of a pressurised pipe and its subsequent whip which may lead to impact of that pipe against another, often at a velocity sufficient to cause major damage.

A dynamic, three-dimensional non-linear finite difference program has been used to model pipe-on-pipe impact and subsequent fracture using an explicit time integration scheme. Both constitutive and geometric non-linearities are accommodated. Fracture is simulated by representing propagating cracks as element-wide bands.

A sample problem demonstrates the effect of a characteristic fracture strain on the serviceability of a pipe after impact.

INTRODUCTION

The design of pressurised pipe systems for nuclear power plants according to the U.S. Atomic Energy Commission Regulatory Guide [1] requires that pipe failure and the consequences of any subsequent whipping motion be considered. It should be assumed that a pipe run may break at any stress raising feature, such as connections to components or other piping, elbows, tees or anchors. Unless restrained adequately, the failed pipe will whip due to the discharge of pressurised fluid and may impact other parts of the piping system which have remained intact. Often the velocity of
impact will be sufficient to cause significant yielding and possibly fracture of one or both pipes. Consequently in regions where a postulated pipe failure would result in damage to other parts of the piping system, present design practice is to provide restraints to prevent any whipping motion.

Most of the associated research work has been concerned with improving the design of the pipe whip restraints and has involved both experimental and numerical investigations [2-4]. The work described here is part of a project which has a different approach to the problem as it aims to develop procedures to design pipes which can withstand impact by a whipping pipe. This approach would simplify the installation of piping systems and hopefully reduce construction costs.

A number of generic calculations of pipe-on-pipe impact have been performed using the computer program PR3D [5]. This is a dynamic three-dimensional computer code which allows for both constitutive and geometric non-linearities as well as contact between interacting bodies. Although fracture as a consequence of impact is an extremely complicated process to model in detail, a simple procedure is used here which reduces the stiffness of elements which are deemed to have fractured.

NUMERICAL PROCEDURE

Each pipe is represented by a mesh of constant strain tetrahedron elements with masses lumped at each node. The mesh is continuously updated during the calculation to represent the deformed shape of the body. Nagtegaal et al. [6] have shown that the number of degrees of freedom per element must exceed the number of constraints imposed by local incompressibility if plastic flow is to be modelled accurately. To do this, a mixed discretisation procedure [7] is employed here. The deviatoric part of the strain tensor is calculated from the mesh of tetrahedron elements while the isotropic strains are referred to a mesh composed of larger elements which each consist of one to five tetrahedrons.

Time integration in PR3D is performed explicitly. With this approach non-linearities can be handled easily as iterations are avoided, although the size of the timestep is limited so that no nodes can communicate physically during one timestep. The analysis therefore proceeds in cycles. Each cycle is typical and performed repeatedly until the timespan of interest has been covered.

At the start of a cycle, the resultant force of each node is computed from the integration of stresses around each node and from external forces and the forces of interaction
with other bodies. The current nodal accelerations are obtained from Newton's second law of motion and their integration yields the nodal velocities and the updated mesh. Incremental strains and rotations can be computed from the incremental displacements. Incremental stresses are then calculated from the constitutive law which employs Jaumann derivatives [8] to account for large deformation effects.

The last step in each cycle involves the computation of the forces of interaction at each node. The algorithm employed in PR3D is able to follow the development of the contact surfaces closely by monitoring the making and breaking of contacts at each node automatically. The contact force at each node is computed as the product of its normal penetration into the neighbouring body and a user-specified contact stiffness. An equal and opposite contact force is applied to the neighbouring body. This is distributed among the nodes of the element face through which the intruding node has penetrated, according to the type of contact (Figure 1).

<table>
<thead>
<tr>
<th>Location of Intersection Point A</th>
<th>Contact Type</th>
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<tbody>
<tr>
<td>node-to-node</td>
<td></td>
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<tr>
<td>node-to-face</td>
<td></td>
</tr>
<tr>
<td>node-to-edge</td>
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</tbody>
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Figure 1: Definition of Contact Type

For a node-to-node contact, the contact force is applied to the nearest node only; for a node-to-edge contact the force is distributed between the two nodes on that edge and for a node-to-face contact, the force is distributed among all three nodes on that face.

SIMULATION OF FRACTURE

To simulate crack growth, a criterion is needed to determine when and where fractures have occurred and a procedure is required to represent the post-fracture response of the material. The latter can be modelled numerically either by allowing sharp cracks to propagate between elements or by representing fractured material as bands of elements
which have reduced stiffness. The fracture process which follows impact is extremely complicated to represent in detail, especially in three-dimensions, as crack initiation, the direction of crack growth, crack branching and arrest need to be considered. Consequently the simpler technique of using crack bands was adopted. This method has been used by Bazant et al. [9] in the analysis of concrete structures and by Wilkins [10] to model the penetration of metal targets by projectiles.

Due to the limited experimental data that was available and the generic nature of this project, a criterion based solely on a critical strain was used. During each cycle, the normal components of the strain tensor in each element are checked. If, in a particular element, one or more components exceed a critical value, which is specified by the program-user, then that element is assumed to have fractured.

If an element has fractured, its deviatoric and tensile stiffness are reduced gradually during the next cycles to zero. Thus the element can sustain only an isotropic compressive stress. A gradual reduction in the stiffness is necessary to limit the effect of numerical instabilities. This procedure is implemented in PR3D using the following algorithm:

if the fracture criterion is satisfied, then in a cycle between time $t$ and $t+\Delta t$,

$$(\sigma)_{t+\Delta t} = \frac{1}{2}(\sigma)_t \quad \text{if} \quad (\sigma)_t > 0 \quad \ldots (1)$$

$$(\sigma)_{t+\Delta t} = (\sigma)_t \quad \text{if} \quad (\sigma)_t < 0 \quad \ldots (2)$$

where $\sigma = \frac{1}{2}(\sigma_{xx} + \sigma_{yy} + \sigma_{zz})$, and

$$(S_{ij})_{t+\Delta t} = \frac{1}{2}(S_{ij})_t$$

where $S_{ij}$ is the deviatoric stress tensor.

SAMPLE PROBLEM AND RESULTS

The problem considered was that of a stationary target pipe, perfectly constrained at both ends, suddenly impacted at equal distances from its extremes by another pipe. This pipe, called the missile pipe, was free at its extremes and the impact took place also at equal distances from both ends. The geometry of each pipe is defined by its length, diameter and thickness. In the case of the target pipe, these were 1000mm, 319mm and 16mm respectively. For the missile pipe, the corresponding dimensions were 1600mm, 395.6mm and 33.5mm.

Both pipes were made from the same steel and were modelled as elastic-perfectly plastic and assumed to obey Von
Mises yield criterion.

Due to symmetry only 1/4 of the problem was analysed with the mesh shown in Figure 2 which consisted of 1148 nodes and 3822 tetrahedron elements.

![Figure 2: Isometric View of the Mesh](image)

The whipping motion of the missile pipe was represented by prescribing an initial velocity of 50 m/sec vertically downwards. The target pipe was stationary prior to impact.

Four calculations were performed using the mesh of Figure 2. In each case the timestep was 1 μsec. The first analysis assumed that the pipe material would not rupture. The other runs assumed critical fracture strains of 10%, 20% and 30% to assess the overall influence of varying rupture strains. Deformed meshes 5 ms after impact are shown in Figures 3-6. Contour lines of equal effective stress are also plotted.
Figure 3: Deformed Mesh after 5 msec - Non Rupturing Material

Figure 4: Deformed Mesh after 5 msec - 10% Critical Rupture Strain
Figure 5: Deformed Mesh after 5 msec - 20% Critical Rupture Strain

Figure 6: Deformed Mesh after 5 msec - 30% Critical Rupture Strain
Time histories of the force of interaction between the two pipes are shown in Figures 7 and 8 for the non-rupturing pipe and the run with a 10% critical rupture strain.

Figure 7: Force of Interaction for Non-Rupturing Material

Figure 8: Force of Interaction for 10% Critical Rupture Strain
CONCLUSION

The runs which used a rupture strain were identical to the non-rupturing analysis up to the point where strains reached the value of the critical rupture strain. Subsequently rupture started and caused the impact load to be smaller than in the non-rupturing case (Figures 7–8). As a consequence of rupture, the extent of the plastic zone in the target pipe was reduced. As elements fractured, further deformations were concentrated in the ruptured elements rather than those undergoing plastic flow (Figures 3–6).

Figures 3–6 also show the effect of increasing rupture strain on the serviceability of the target pipe following impact. In the runs involving critical fracture strains of 10% and 20%, the target pipe has suffered major damage and has clearly failed. With the 30% fracture strain, only limited cracking occurred and the overall integrity of the target pipe was maintained.
REFERENCES

[1] U.S. ATOMIC ENERGY COMMISSION REGULATORY GUIDE 1.46
'Protection against Pipe Whip inside Containment'

[2] IKONEN, K., KUKKOLA, T. and KANGAS, M.
'Local Crush Rigidity of Pipes; Experiments and
Application to Pipe Whip Restraint Design'
Paper F6/4, 5th International Conference on Structural

'Study of Pipe Rupture Dynamics: Aquitaine II Program'
Paper F8/4, 6th International Conference on Structural

[4] UEDA, S., KURIHARA, R., ISOZAKI, T., MIYAZAKI, N.,
KATO, R., MIYAZONO, S., and SAITO, K.
'Experimental and Analytical Results of 4 inch Pipe
Whip Tests under BWR Conditions'
Paper F6/3, 6th International Conference on Structural

[5] PRINCIPIA MECHANICA LIMITED
'PR3D Documentation'
Reports Nos. PR-TN-4 TO PR-TN-7, Version 3P-2, June
1983.

'On Numerically Accurate Finite Element Solutions in
the Fully Plastic Range'

'Mixed Discretisation Procedure for Accurate Modelling
of Plastic Collapse'

[8] PRAGER, W.
'An Elementary Discussion of Definitions of Stress
Rate'

'Fracture Mechanics of Reinforced Concrete'

[10] WILKINS, M.L.
'Mechanics of Penetration and Perforation'