THREE-DIMENSIONAL PILE-SOIL-PILE INTERACTION ANALYSIS FOR PILE GROUPS

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ABSTRACT

A procedure is presented for conducting dynamic analyses of dense pile groups based on the complex response method in the frequency domain. The soil is treated as a linear viscoelastic material with hysteretic damping. A three-dimensional soil dynamic stiffness matrix relating points corresponding to pile nodes is derived from a geometrically axisymmetric finite element model. Solutions are obtained for loads in two Fourier harmonics, simulating horizontal and vertical forces applied at points along the axis of the model. The responses to these forces represent the complex flexibility coefficients of the soil. The total soil complex flexibility matrix is evaluated by superimposing the displacement field for all pile nodes. The inverse of the flexibility matrix gives the dynamic stiffness matrix of the soil. Pile dynamic stiffnesses are then superimposed to those of the soil system and all degrees of freedom except those associated with pile heads are eliminated by a simple reduction procedure. The resulting impedance functions fully incorporate pile-soil-pile interaction. The three-dimensional structural analysis then follows in the traditional fashion. To demonstrate the effects of the pile-soil-pile interaction, the dynamic stiffness and damping coefficients of a foundation consisting of 2 x 2 piles are calculated and compared for different pile radii, distance between piles and soil profiles.
1. INTRODUCTION

Piles have been used for many years to improve some of the characteristics of structural foundations in weak soils, particularly in respect of their stiffness properties and bearing capacity. In exchange for these advantages, pile foundations are usually more complex and costly. An important consideration when assessing the desirability of a pile foundation for nuclear structures is the increased difficulty of the necessary design and safety calculations. The analysis of the soil-structure and pile-soil-pile interaction should be approached in principle as a three-dimensional non-linear problem. Determination of the response of such a system to an earthquake would require great computational effort, firstly because the idealisations used in such analysis would result in a large number of degrees of freedom, and secondly, the solution would have to be evaluated successively at many different times. This may be possible with a powerful computer and no constraints on computing time. However, in practice, some simplifying approximations must be made. The state-of-the-art opens two major possibilities:

- The non-linearity of the pile-soil interaction is accounted for usually by means of the so-called p-y curves or non-linear subgrade reaction moduli, Penzien et al [1]. The pile-soil-pile interaction must then be neglected. This method is very popular in analysis of offshore pile structures.
- Pile-soil-pile interaction is duly accounted for by using superposition methods. However, the non-linearity of soil behaviour must be neglected.

The second method is based on single pile analyses. It can rely on continuous models and theory of elasticity (Novak [2]), as well as in discrete models, such as those used in the finite element method (Blaney et al [3]). These can be generalised then by superposition for the analysis of pile groups.

None of the above routes is entirely satisfactory. But as pile spacings under nuclear facilities and particularly under the reactor building are usually very small (3 or 4 pile diameters), the
pile-soil-pile interaction should be taken into account. In this light, the second possibility appears more realistic. Based on this reasoning, a computer program was developed in which pile-soil-pile interaction and radiation damping are properly modelled at the price of a linearly viscoelastic soil with hysteretic damping. Such procedure allows carrying out the analysis by the complex response method in the frequency domain. The approach selected is a simplified and possibly improved version of that by Wolf and von Arx [4]. The complete analysis includes the following steps:

- Fourier transform analysis of the seismic input motion
- Calculation of the complex impedance functions for the basement for the set of frequencies of interest
- Calculation of the transfer functions of the seismic input motion
- Dynamic analysis of the structure and the soil-pile foundation
- Calculation of pile displacements and stresses due to the obtained basement motion
- Inverse Fourier transform of the results

Mathematical formulation of the complex response method in the frequency domain is well known and may be found in many textbooks, e.g. Clough and Penzien [5]. The calculation of the impedance and transfer functions for a rigid basement by the finite element method is presented in the following section.

2. CALCULATION OF THE IMPEDANCE AND TRANSFER FUNCTIONS

The calculation incorporated in the program for Pile-soil-pile INTERaction (PINTER) is described here due to its special interest. Since the analysis is linear and utilises the principle of superposition, the soil is considered first. Displacements \( r_s \) at points in the soil medium are related to the forces acting at those points \( R_S \) by a simple relationship

\[
R_S = S_S r_s \tag{1}
\]

where \( S_s \) is the complex dynamic stiffness matrix of the soil. Note that in the complex response method all displacements and forces are complex
amplitudes and should be multiplied by $e^{i\omega t}$, where $\omega$ is the frequency of excitation. This has been omitted here for simplicity. The form of the complex dynamic stiffness matrix is as follows

$$
S = -\omega^2 M + (1+2iD) K
$$

where $M = \text{mass matrix (real)}$

$K = \text{stiffness matrix (real)}$

$\omega = \text{circular frequency}$

$D = \text{ratio of hysteretic damping}$

Since this is a general expression all indices are omitted. To obtain the complex soil stiffness $S_s$, different methods may be applied. With no constraints on computer time and capacity, a straightforward finite element discretisation could be applied. However, to reduce the size of the problem, it is possible to utilise a two-dimensional solution to produce a fully three-dimensional complex flexibility matrix of the soil which inverted gives the desired stiffness. Therefore the soil is idealised as an axisymmetric continuum of isotropic viscoelastic material which consists of horizontal layers. The soil properties may vary with depth, but remain constant within each layer. The finite element discretisation of the soil makes use of a four-noded toroidal element. The variation of the nodal parameters around the circumference of the element is described by the Fourier series. Thus utilising a series of two-dimensional solutions which are decoupled, full three-dimensional effects are obtained. The boundary of this axisymmetric model may be fixed or non-reflecting (viscous) but a more complicated consistent boundary can be included as well. The presence of a single pile at the centre of the axisymmetric mesh is simulated by forcing the cross-section at imaginary pile nodes to be rigid. Unit translational force (in the first harmonic) and unit vertical force (in the zero harmonic) are applied in turn to all nodes of the imaginary pile. Hence the calculated displacement amplitudes represent the complex flexibility coefficients. For each pile in a group, these flexibility coefficients are calculated for all pile nodes by simple interpolation. This yields the complex soil flexibility matrix which relates forces and displacements as follows

$$
\tau_s = F_s^T R_s
$$
The complex stiffness matrix of the soil $S_s$ is obtained by inversion of this flexibility matrix. Note that there are no restrictions on the position of pile nodes in the program; piles may be vertical or battered, floating or end-bearing. The more generalised version of the program calculates the soil flexibility matrix for any given set of points and this may be used to calculate impedance functions of embedded foundations. In other words, eqs. (1) and (3) relate soil displacements and forces for a set of points which, for a pile foundation, happen to coincide with pile axes.

The soil displacements $x_s$ are relative displacements, i.e. the total displacements are

$$x = x_s + x_f$$  (4)

where $x_f$ are free-field displacements. Denoting forces acting on pile heads by $R_h$, the equilibrium equation for pile foundation follows

$$S_p x + R_s = R_h$$  (5)

where $S_p$ is the complex dynamic stiffness matrix of all piles. Matrix $S_p$ is of the form given in eq. (2). The piles are modelled by a two-noded three-dimensional beam element with three displacements and three rotations at each node. Since the soil stiffness coefficients at each node correspond to three displacements, to avoid incompatibility, it has been assumed here that there were no moments applied at pile nodes other than pile heads. This enables all rotations except those of pile heads to be eliminated by simple static condensation without any loss of accuracy in pile performance. Substituting eqs. (1) and (4) into eq. (5) yields

$$(S_p + S_s) x = R_h + S_s x_f$$  (6)

The second term on the right-hand side corresponds to the forces produced by a free-field motion. It should be noted at this stage that cavities may also be included in the calculation by substituting $S_s$ with

$$S_{sc} = S_s - S_c$$  (7)
where \( S_c \) is the complex dynamic stiffness matrix of cavities. The static condensation is now performed to eliminate all variables which do not correspond to pile heads. This results in the following

\[
G_h \mathbf{r}_h = \mathbf{R}_h + \mathbf{R}_f \tag{8}
\]

where \( G_h \) is the complex dynamic stiffness matrix for pile heads and \( \mathbf{R}_f \) is the vector of the amplitudes of the reactions at pile heads induced by a free-field motion and by fixing pile heads (vectors \( S_s \mathbf{r}_f \) reduced to pile heads).

The dynamic stiffness matrix of pile heads may now be coupled to the elastic basement or superstructure. If the basement is assumed to be rigid then pile head displacements \( \mathbf{r}_h \) may be expressed in terms of the basement displacements \( \mathbf{r}_o \) as follows

\[
\mathbf{r}_h = \mathbf{T} \mathbf{r}_o \tag{9}
\]

where \( \mathbf{T} \) is a transformation matrix. The displacement transformation eq.(9) is now introduced into eq.(8) to give

\[
\mathbf{Q}_o \mathbf{r}_o = \mathbf{Z}_h + \mathbf{Z}_f \tag{10}
\]

where \( \mathbf{Q}_o \) is a 6 x 6 impedance matrix of the basement defined as

\[
\mathbf{Q}_o = \mathbf{T}^t G_h \mathbf{T} \tag{11}
\]

and

\[
\mathbf{Z}_h = \mathbf{T}^t \mathbf{R}_h \tag{12a}
\]

\[
\mathbf{Z}_f = \mathbf{T}^t \mathbf{R}_f \tag{12b}
\]

The transfer functions to the basement are calculated as follows

\[
\mathbf{r}_o = \mathbf{Q}_o^{-1} (\mathbf{Z}_h + \mathbf{Z}_f) \tag{13}
\]

The impedance functions eq.(11) and the forces eq.(12) are coupled to the superstructure and the solution is completed. Once the motion of the basement and the superstructure is determined it is necessary to calculate forces and moments on the piles. The program for this task is called
stiffness coefficients are presented in Fig.3. The notation is the same as in ref.[6], and [8], therefore \( k_{xx} \) is the stiffness coefficient (real part) for a pile group when pile-soil-pile interaction is taken into consideration, \( k_{xx}^{s} \) stands for a single pile, hence \( k_{xx}^{s} \) denotes the stiffness coefficient for a pile group neglecting pile-soil-pile interaction. The damping coefficients (imaginary part) are denoted by \( c_{xx}^{s} \), \( c_{xx} \) and \( c_{xx}^{s} \) respectively. It is interesting to note the effect of the pile stiffness on the pile-soil-pile interaction. There is a remarkable difference between piles of \( 2r=0.32m \) and \( 2r=1.30m \); this was not reported by Wolf [6], whose corresponding curves nearly coincide.

For a particular pile of a diameter \( 2r=1.3m \) the interaction effects may be neglected for a distance-to-diameter ratio exceeding 100 for a vibration in the horizontal direction. Intuitively this ratio seems to be very high and some kind of 'non-linearity' proposed by Novak [9], would be desirable. To show the influence of the soil properties for the pile group of a diameter \( 2r=1.30m \), the calculation is repeated for a weak soil.

Pile properties are the same as before, while the soil layer is of uniform properties: shear modulus \( G=60\, \text{MN/m}^2 \), mass density \( \rho=1.5\, \text{Mg/m}^3 \), Poisson’s ratio \( v=0.4 \), ratio of the hysteretic damping \( D=0.05 \). The finite element mesh and all the other data is the same as before. The results are given in Fig.4.

From the simple example presented here the following conclusion may be suggested:

- Interaction effects are stronger for softer soil media
- Interaction effects are stronger for more flexible piles
- Interaction effects are negligible for large pile distance to diameter ratios
- Radiation damping, in general, increases with pile distance to diameter ratios
PILFOR and basically follows the described procedure in the reversed order.

3. EXAMPLE

The procedure of the previous section is applied to the calculation of dynamic stiffness coefficients for a 2 x 2 pile foundation for different pile radii, distance between piles and soil properties.

A stratified layer of soil of a depth of 34 m resting on rigid bedrock is analysed first. The material properties are listed in Table I (following Wolf [6]). The ratio of hysteretic damping for all layers is 0.05. Piles are made of concrete with a modulus of elasticity $E=30 \text{GN/m}^2$, a mass density $\rho=2.5 \text{Mg/m}^3$, a ratio of hysteretic damping $D=0$ and a length $h=34 \text{m}$. Three pile diameters are considered: $2r=0.32 \text{m}$, $1.30 \text{m}$ and $1.80 \text{m}$. The distance $b$ (Fig.1) is varied. The finite element idealisation of soil layer is presented in Fig.2. The overall radius of the axisymmetric soil mesh is $300 \text{m}$, soil is bonded to the bedrock and the far-field is represented by the viscous boundary of Lysmer and Kuhlemeyer [7]. The frequency of excitation is $4 \text{Hz}$. The dynamic

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>Property</th>
<th>Shear Modulus (GN/m²)</th>
<th>Mass Density (Mg/m³)</th>
<th>Poisson’s Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>SAND</td>
<td>0.435</td>
<td>1.95</td>
<td>0.42</td>
</tr>
<tr>
<td>-4.5</td>
<td>CLAY</td>
<td>0.027</td>
<td>1.68</td>
<td>0.49</td>
</tr>
<tr>
<td>-7.5</td>
<td>TRANSITION</td>
<td>0.944</td>
<td>2.10</td>
<td>0.35</td>
</tr>
<tr>
<td>-11.5</td>
<td>RESIDUAL 1</td>
<td>1.022</td>
<td>2.05</td>
<td>0.36</td>
</tr>
<tr>
<td>-20.0</td>
<td>RESIDUAL 2</td>
<td>1.556</td>
<td>2.05</td>
<td>0.34</td>
</tr>
<tr>
<td>-27.0</td>
<td>WEATHERED</td>
<td>3.018</td>
<td>2.40</td>
<td>0.43</td>
</tr>
<tr>
<td>-34.0</td>
<td>ROCK</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
4. REFERENCES


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Fig. 4a

HORIZONTAL STIFFNESS COEFFICIENTS AS A FUNCTION OF DISTANCE BETWEEN PILES FOR TWO SOIL PROFILES

PILE DIAMETER $D = 0.05 m$

SOIL PROFILE TABLE 3

UNIFORM SOIL

$S = 20 kN/m^2$

$\rho = 1.5 Mg/m^3$

$e = 0.4$

$D = 0.05$
4. REFERENCES


