A NUMERICAL AND EXPERIMENTAL STUDY OF DEEP ELASTOPLASTIC INDENTATION

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ABSTRACT

Indentation experiments have been performed with a square-ended punch (5mm x 5 mm) in a 6 mm thick plate of soft copper, near a free edge. Load-penetration/free surface distortion data have been measured and compared with a numerical simulation which used the computer program PR3D.

1. INTRODUCTION

Principia Mechanica Ltd. regularly tackle deformation problems in soil mechanics using a time-marching dynamic elastoplastic numerical analysis. Plastic deformation of porous media involves formidable problems with pressure-dependent yield criteria and non-associated flow rules. Plastic deformation of metals on the other hand is usually considered to be pressure-independent and the principle of normality applies. Even so the computer solution of rigid-plastic problems, and certainly elastoplastic problems, in metalforming plasticity can be difficult. It occurred to one of us that if Principia’s methods are capable of solving soil mechanics problems, then they surely could be applied to metalforming. This paper presents the first exploratory results from such an exercise. The problem chosen is that of a square-ended punch indenting a thin plate of a highly work-hardening material near a free edge.

2. THE NUMERICAL APPROACH

The analysis is 3-dimensional and based on a Lagrangian finite difference scheme, with differences being central both in space and time. The mass of the continuum is lumped at grid points and this lumped mass remains constant during the analysis in spite of large deformations because of the Lagrangian character of the procedure, i.e. the mesh moves with the deforming body.
Two basic sets of equations have to be satisfied in problems of mechanical deformation: the constitutive equations and the conservation of momentum equations. The constitutive equations are formulated incrementally and are always applied in an explicit fashion, that is, increments of the stress tensor are computed as a function of strain increments, strains, strain-rates, etc. But this relationship is never inverted, thus avoiding singularities and instabilities in softening materials.

The full momentum equations are used without any simplifications, even for static problems. If the conditions imposed generate a sufficiently slow problem, inertia forces will be negligible and the solution will be independent of the density; however, the full dynamic equations will still be adequate to reach the solution. The use of a dynamic approach when analysing static problems has several advantages: (a) it acts in a way similar to a static incremental approach with small increments, in that constitutive and geometrical non-linearities are closely approached in an incremental fashion (b) it provides accurate solutions to path-dependent problems, as needed for example in elasto-plastic modelling (c) the releases of energy which occur during constitutive softening, material instability (some bifurcation problems) or geometrical instability (buckling, etc.) are handled in the natural way: the energy released starts travelling along the body in the form of waves without giving rise to numerical instabilities.

In solving a problem numerically, space and time must be discretised into small domains. There is a choice in selecting their characteristics, primarily the size of the elements of space and time discretisations. Simple (constant-strain) space elements can be used. However the same resolution can be achieved with more complex (higher order) but fewer elements. The same applies to the time increment used for integration of the momentum equations. A small timestep (explicit integration) uncouples the equation of motion. A large one (implicit integration) requires a smaller number of more complex timesteps.

The simpler space elements together with smaller timesteps allow for greater generality of treatment, constitutive behaviour, etc. However, these may result in greater computing requirements for problems where the added generality is not needed, such as in elastic, small-strain calculations. The options can be described simply in each case as performing fewer but more complex calculations, or more but simpler calculations. The problems of adequate space discretisation are further discussed in the next section.

3. MIXED DISCRETISATION

Nagtegaal et al [1] pointed out that the number of degrees of freedom per space element must be greater than the number of
strains and rotations in an incremental formulation:

\[
\Delta \varepsilon_{ij} = \frac{1}{2} (\dot{u}_{ij} + \dot{u}_{ji}) \Delta t
\]

\[
\Delta \theta_{ij} = \frac{1}{2} (\dot{u}_{ij} - \dot{u}_{ji}) \Delta t
\]

\[\ldots(3)\]

where \(\varepsilon_{ij}\), \(\theta_{ij}\) are the strain and rotation tensors, respectively,

The rotation tensor is required for implementing the Jaumann formulation which must apply to all variables referred to a Lagrangian frame, in particular the stress and strain tensors. The strain and strain increment tensors are used to enter the constitutive laws which, in the present case, simply consist of an elastoplastic model with power-law isotropic hardening. The elastic part is calculated incrementally:

\[
\Delta \sigma_{ij} = \lambda \Delta \varepsilon_{kk} \delta_{ij} + 2\mu \Delta \varepsilon_{ij}
\]

\[\ldots(4)\]

where \(\sigma_{ij}\) is the stress tensor

\(\lambda, \mu\) are the Lamé constants

The yield criterion is expressed as:

\[
\bar{\tau} \leq A \bar{\varepsilon}^n
\]

\[\ldots(5)\]

where \(\bar{\tau}, \bar{\varepsilon}\) are the effective stress and strain, respectively and \(A, n\) are material properties.

The stresses thus produced are then used to enter once again eq. 1 and restart the computational cycle. The procedure is stable as long as the timestep is sufficiently small such that the equations of motion of nodes are uncoupled, that is, as long as the Courant condition is fulfilled. The incremental approach, Jaumann formulation and updated geometry guarantee accurate treatment of large displacements, strains and rotations.

The formulation described is embodied in a commercially available computer program, PR3D [3], which was used for the calculations presented below.

5. SAMPLE PROBLEM AND EXPERIMENTAL RESULTS

The numerical procedure has been utilised to analyse a deep indentation problem. A rough rigid die, measuring 5mm x 5mm, located 5mm away from one edge of a 6mm thick copper plate slowly indents the plate which rests on a frictionless surface, Fig.1. All other boundaries, except for the area in contact with the die, are free to move in any direction. A total of 1330 nodes and 5832 tetrahedrons were used to model the required half of the problem. Owing to the symmetry condition, nodes on the symmetry plane are not allowed to leave it.

The values of the material properties needed in eqs. 4 and 5 were obtained from independent experiments as,

\[
\lambda = 6.679 \times 10^{10} \text{ N/m}^2 \quad \mu = 4.519 \times 10^{10} \text{ N/m}^2
\]
\[ A = 4.6 \times 10^8 \text{ N/m}^2 \quad n = 0.38 \]

**Fig. 1**

Since the results of a static problem are independent of the value of the density of the material, density scaling by six orders of magnitude was used in order to increase the size of the allowable timestep and thus expedite the calculations. The complete problem, reaching a penetration of 3.125\text{mm} (5/8ths of the die dimension) used 750 timesteps.

Figure 2 shows the force-indentation curve obtained from the analysis. The overall deformation pattern can be assessed...
in Figure 3, where groups of six tetrahedrons have been plotted as a hexahedron for easier visualisation.

In the complementary experiments, a low friction platen for the base of the copper block was achieved with copious supplies of lubricating oil. The punch and top surface of the copper plate were degreased, but the punch was not deliberately roughened. The experimental load-penetration plot (corrected for machine stiffness) is superimposed in Fig. 2; similar curves were obtained with interrupted or continuous loading. The data agree in general terms with the theory, but are about 10% low. This is because we are not really comparing like with like since the punch is not perfectly rough as assumed in the calculations. The overall distortion of the block also agrees quantitatively with the sense of Fig. 3 and microhardness indentations have been compared with computed lines of equal effective stress on longitudinal and transverse sections, Figures 4 and 5.

Given the slight differences in interfacial conditions at the punch as between the computations and experiments, the overall agreement is satisfactory.

6 CONCLUSIONS

The exploratory calculations and experiments described in this paper demonstrate that the PR3D program seems well-suited for tackling difficult elastoplastic metal deformation problems.

REFERENCES


Fig. 4 Computed Lines of Equal Effective Stress (10$\Delta$N/m$^2$) on Longitudinal Section for the punch penetration shown in Fig. 3.
FIG. 5  Computed lines of equal effective stresses (10^6 N/m^2)

shown in Figure 3.

on transverse sections for the punch penetration.